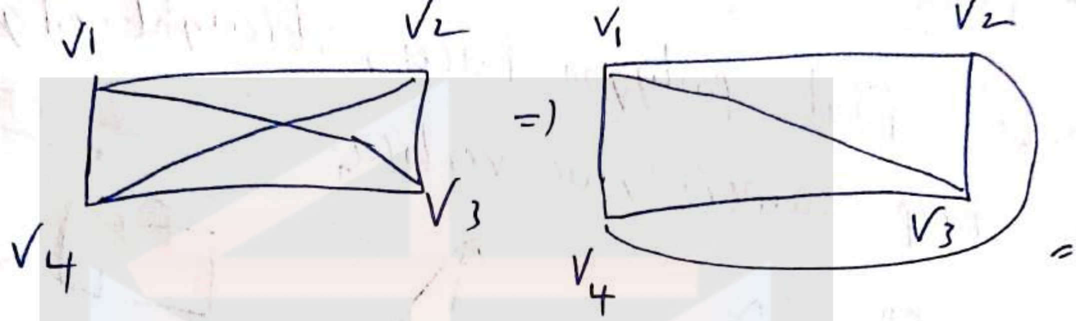
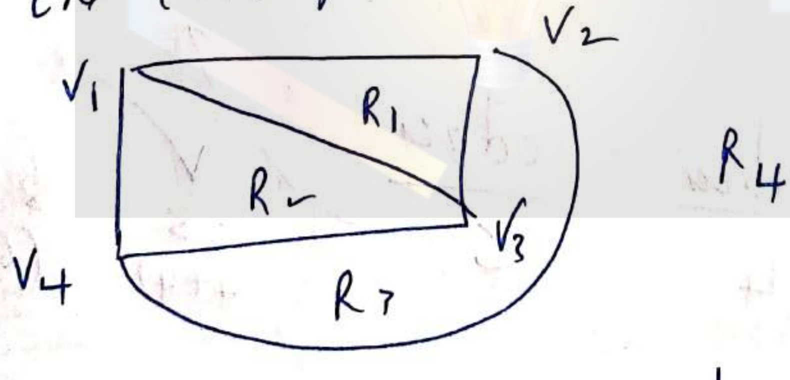
Unit V Planar graph & Directed graph

→ **Planar Graph:** It is a graph that can be drawn in the plane without any edges crossing

ex



→ **Faces / region:** A planar graph can be divided into the contiguous regions called faces



→ **Euler's Formula:** Let G be a connected planar simple graph with e edges & v vertices. Let r be the no. of regions

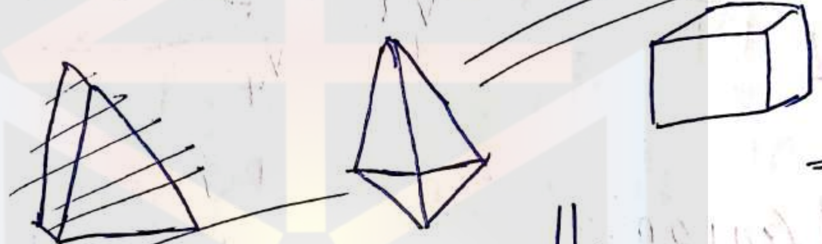
$r = e - v + 2$

$r = 6 - 4 + 2$

$= \underline{\underline{4}}$ Hence proved

→ polyhedrons :- Polyhedrons are 3 D shapes with flat polygon faces, straight edges & sharp corners, or vertices

ex



$\Downarrow$

These do follow $r = e - v + 2$

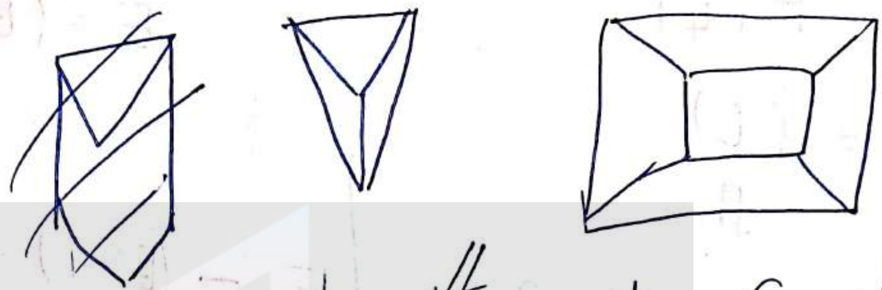
$r + v = e + 2$

Faces/region	Vertices	edges	
4	4	6	$8 = 8$ ✓ $(4+4)$ $(6+2)$
6	8	12	$6 + 8 = 12 + 2$ $(14 = 14)$ ✓

hence proved

∴ any planar graph / which can be draw without cross .. can be applied th-

$r = e - v + 2$

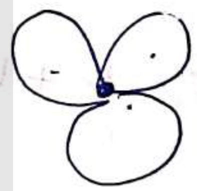
⟹ 

⇓ Follow euler formula

<u>Proff</u>

asum vertex = 1

$v = 1$



$V = 1$
$E = 3$
$r = 4$

⇓

$r = E + 1$

edge will be loops

& face / region will be edge + 1

$r = E + 1$

$\therefore \frac{F - E}{1} + \frac{V}{1} = 2$ hence proved for $v = 1$

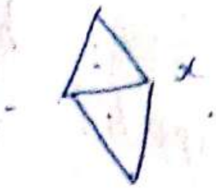
For $v = n$

$F - E + V = 2$

$v = n + 1$

$F - E + V = 2$

ex:



$V = (n)(4)$

$F = F(4)$

$E = (E)$

$\Downarrow$

$F - E + V = 2$

$F - E + V = 2$ ✓

$x - y = x$

$V = (n+1)(4+1)/(3)$

$F = F(4)$

$E = (E-1)$

$F - (E-1) + (V+1) = 2$

$F - E \cancel{-1} + V \cancel{+1} = 2$

$F - E + V = 2$ ✓

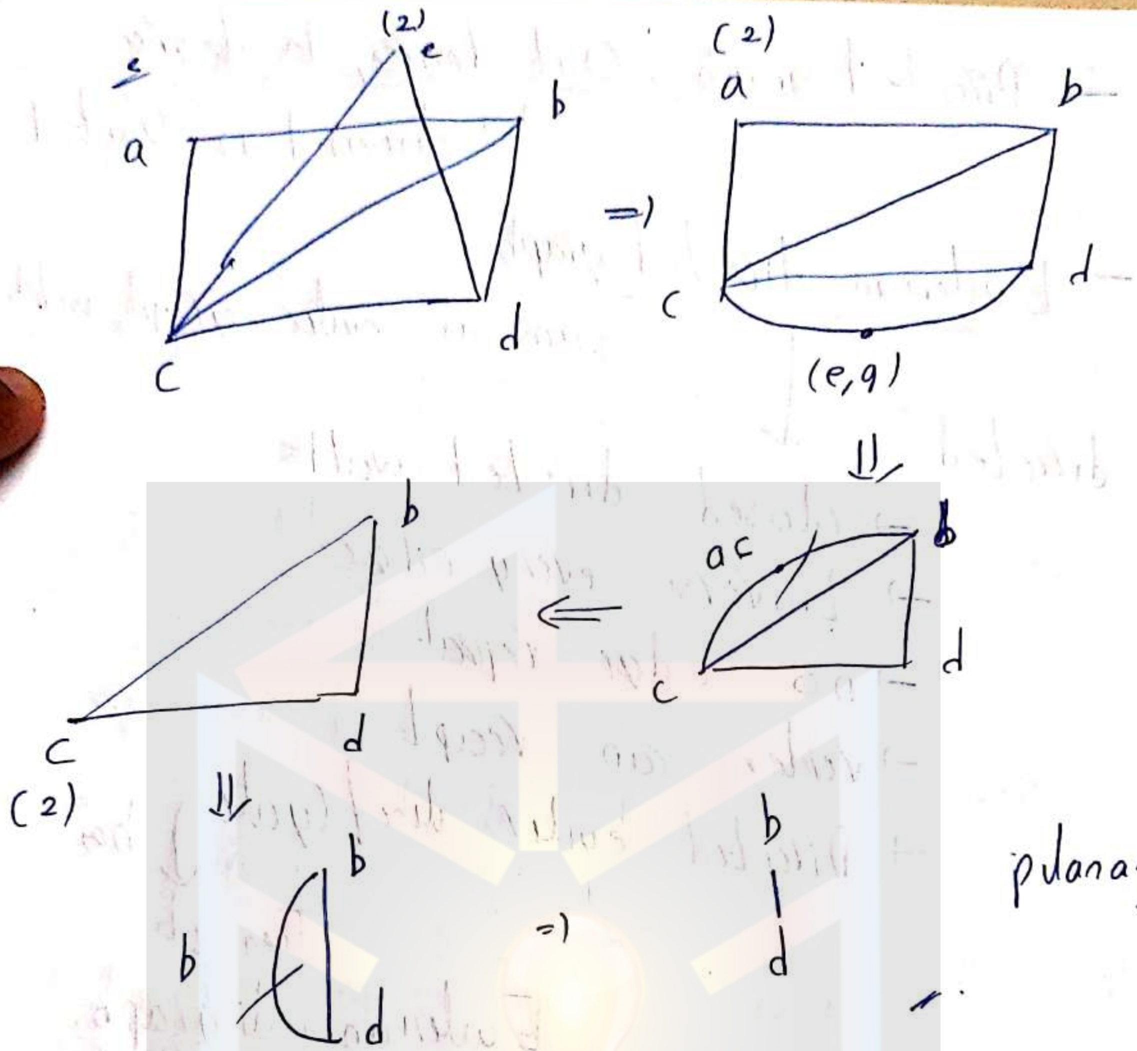
Hence proved

→ **Planarity Testing algorithm:**

→ If graph is disconnected then it has several components, take 1 at a time

→ If graph has parallel edges or self loop, remove it

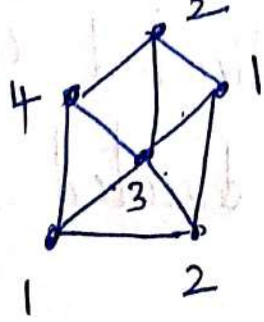
→ If their are vertex in the graph of degree 2, we will merge them



planar

→ 5 Color Theorem:

It states that any simple, planar graph can be 5 colorable (colored using 5 colors). (5 or less)



4 colors

[Proper proof not found if u have plan share)

→ Directed graph, Out degree, In degree

↳ discussed in Unit 1

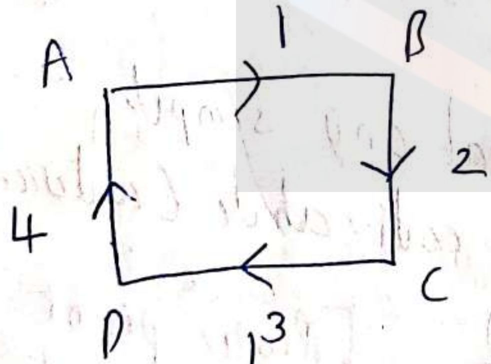
→ Eulerian directed graph

same as euler graph with directed

→ closed directed walk
→ traverse every edge
→ no edge repeat
→ vertex can repeat
→ Directed euler walk / cycle } has

Then it Eulerian digraph.

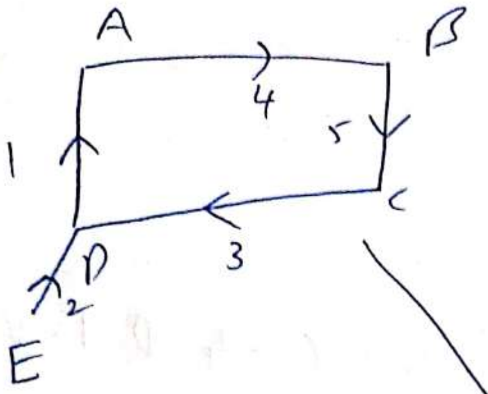
eg



A - 1 - B - 2 - C - 3 - D - 4 - A

closed
no edge repeat ✓
directed closed walk

eulerian directed graph.



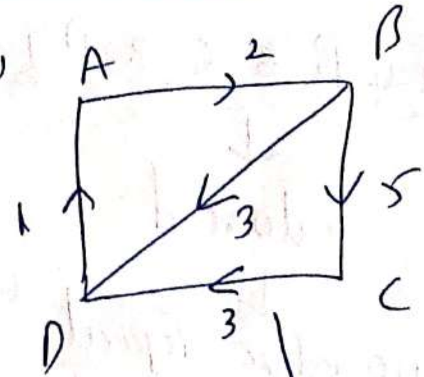
A 4 B 5 C 3 D 1 A
↓
closed
↓
no edge repeate.
but not visited all edges
not directed euler cycle

E 2 D 1 A 4 B 5 C 3 D
never starting & ending will be same if we pass thru E 2 D.

not eulerian directed graph.

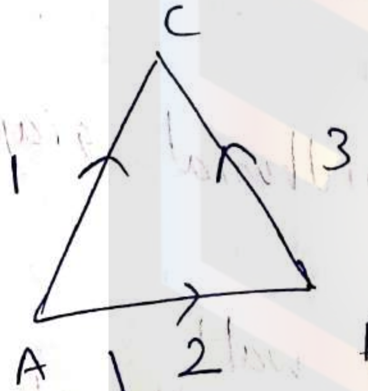
→ Hamiltonian Diagraphs
same as hamiltonial graph with direction.
→ closed directed walk
→ traverse exactly once through vertex.
→ closed.
→ edge can't repeat
→ Directed hamiltonian cycle
then

eg



A 2B 3D 1 A
closed walk
(C missing)
not hamiltanian ~~graph~~ cycle

A-2-B-5-C-4-D-1-A
closed walk
visited all
hamiltonian cycle
∴ hamiltonian graph



A - 2 B 3 C 1 A
closed walk
order wrong X
not hamiltonian cycle

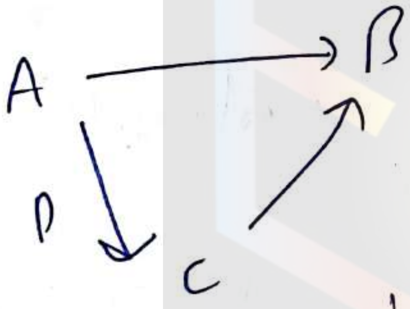
not possible
hence not hamiltonian graph

→ Tournament :

A tournament T is an oriented (directed) completed graph

K_2 K_3 K_4

losers get the arrow
winner get the end



A v/s B
↑
won

	win	loss
A	2	0
B	0	2
C	1	1