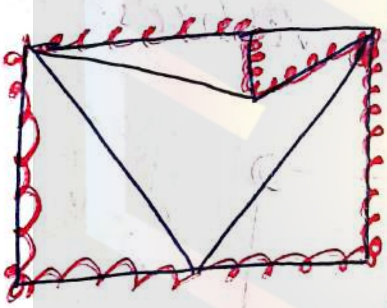
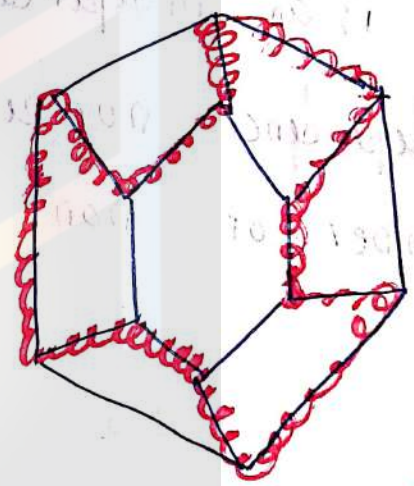
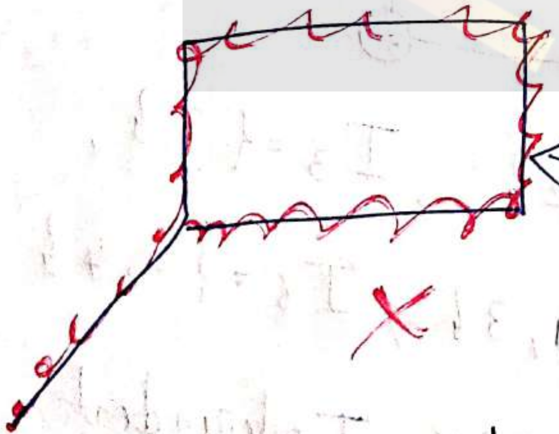
Unit - III hamiltonian graphs :

→ **Hamiltonian circuit/cycle** :- in a connected graph is defined as a closed walk that traverse every vertex of G exactly once. except starting/ending vertex

→ **Hamiltonian graph** :- A graph G is said to be hamiltonian if it has a hamiltonian circuit/cycle.







hamiltonian path :- is open walk.

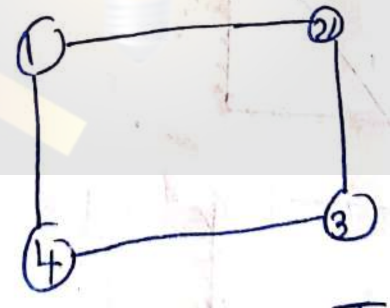
ex: Travelling salesman problem uses it

→ Independent Sets :- /matching

→ A set of vertices I is called independent set if no two vertices in set I are adjacent to each other or in other words the set of non-adjacent vertices is called independent set/stable set.

→ The parameter $\alpha_0(G) = \max\{|I| :$ I is an independent set in G $\}$ is called independence number of G, i.e the max number of non adjacent vertices.

e.g

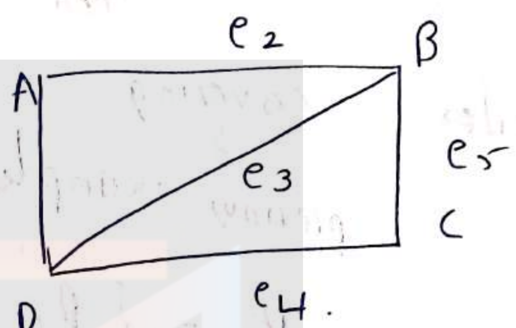


$I_1 = \{1\}$, $I_2 = \{2\}$, $I_3 = \{3\}$,
$I_4 = \{4\}$, $I_5 = \{1, 3\}$, $I_6 = \{2, 4\}$

$\alpha_0(G) = 2 \leftarrow$ max Independent number.

→ Covering :-

→ **Vertex covering:** A subset of V (vertices) is called a vertex covering of graph G if every edge of G is incident with or covered by a vertex in subset of V



$V = \{A, B, C, D\}$

$V_1 = \{B, D\}$

B: e_2, e_3, e_5 D: e_1, e_3, e_4

$\Downarrow$

e_1, e_2, e_3, e_4, e_5

$\therefore$ $\{B, D\}$ is vertex covering.

$V_2 = \{A, B, C\}$ ✓

$V_3 = \{A, C\}$ → not vertex covering

A: e_2, e_1 C: e_4, e_5

e_3 missing hence

minimal vertex covering:

if all the vertices in the set are mandatory to maintain vertex covering. Then that set is min vertex covering.

previous examples

$V_1 = \{B, D\} \rightarrow$ MVC ✓

$V_2 = \{A, B, C\} \rightarrow$ MVC ✓

assume $V_5 = \{A, B, C, D\} \rightarrow$ Not MVC

↑ we can remove any one to make MVC

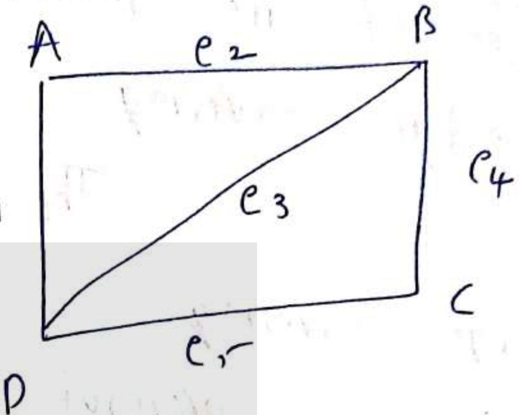
$\beta(G)$ indicates cost of MVC

minimum vertex covering.

$\rightarrow$ $\underline{\underline{2}}$ in above example

→ Line / Edge covering :-

A subset of edges is called a edge covering if every vertex of G is incident with atleast one edge.

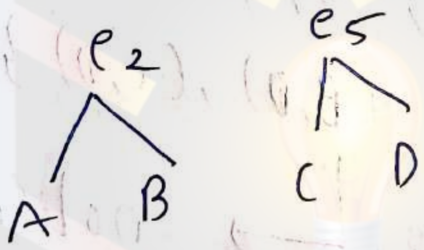


$E = \{e_1, e_2, e_3, e_4, e_5\}$

$\Updownarrow$ or

$E = \{(A,D), (A,B), (B,D), (B,C), (C,D)\}$

$E_1 = \{(A,B), (C,D)\}$ ✓



$E_2 = \{(A,D), (B,C)\}$ ✓

$E_3 = \{(A,B), (B,D), (C,D)\}$ ✓

$E_4 = \{(A,B), (B,C) (B,D)\}$ ✓

$E_5 = \{(A,B), (B,D)\}$ X

c missing

minimal edge covering:-

if all the edges in the set are mandatory to maintain edge covering.

Then that set is minimal edge covering

previous example

$E_1 = \{(A, B), (C, D)\} \rightarrow$ MEC

$E_2 = \{(A, D), (B, C)\} \rightarrow$ MEC

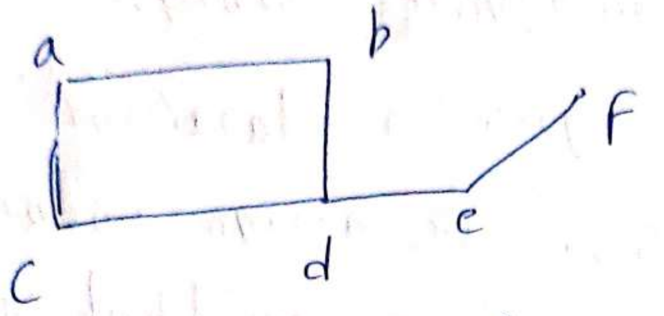
$E_3 = \{(A, B), (B, D), (C, D)\}$

$\rightarrow$ not mandatory can be removed

$\therefore$ minimum edge covering is the count

$\Rightarrow$ 2.

Note → $\boxed{\text{matching is nothing but Independent set}}$

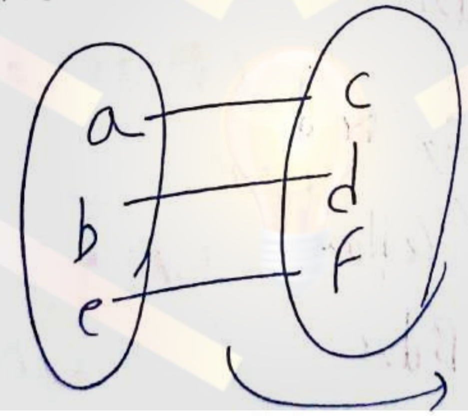


$\{ac, eF\} = M_1$ (non maximal)

$M_2 = \{ac, eF, bd\}$ perfect/complete matching

maximal

maximum



bipartite graph