

Unit II

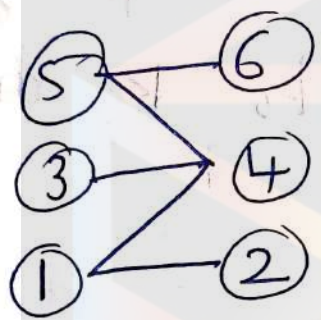
Eulerian graph & special graphs

→ Bipartite Graph:

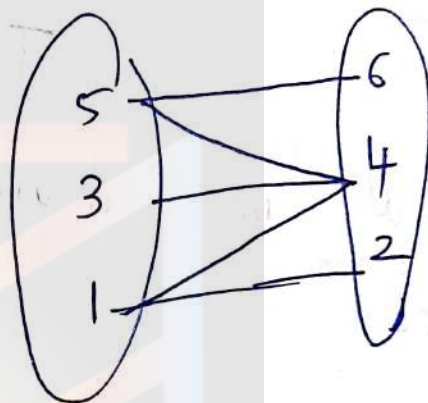
→ It is a simple graph in which the set of vertices can be partitioned into two sets

X & Y

ex



⇒



Bipartite graph

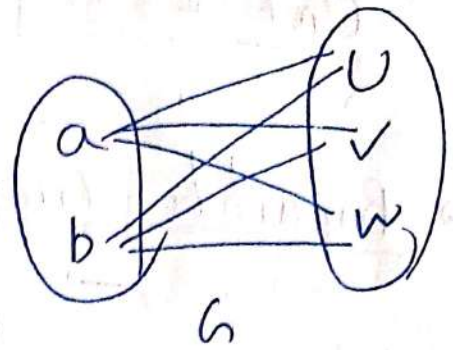
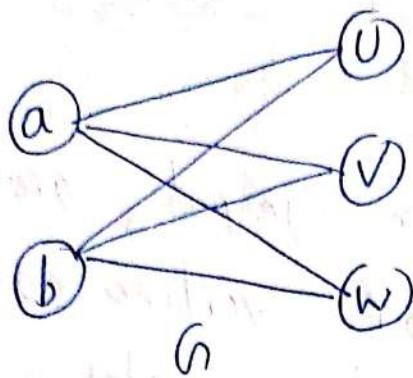
Complete Bipartite graph

→ It is a bipartite graph in which there is an edge b/w every vertex x & every vertex y

\uparrow
 $x \in X$

\uparrow
 $y \in Y$

ex



G

G

complete Bipartite.

→ k - Partite Graph:-

It is a k partite graph we will be dividing into k disjoint subsets.

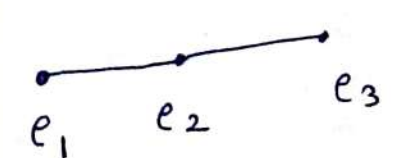
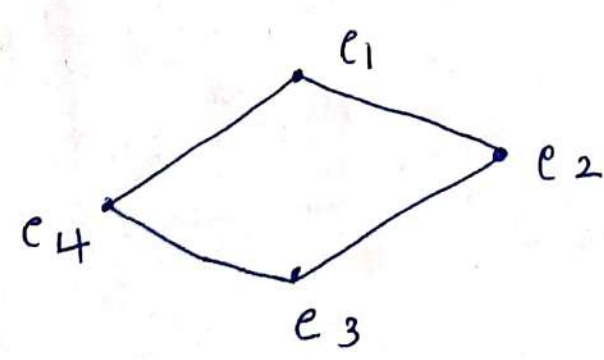
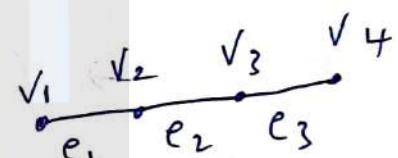
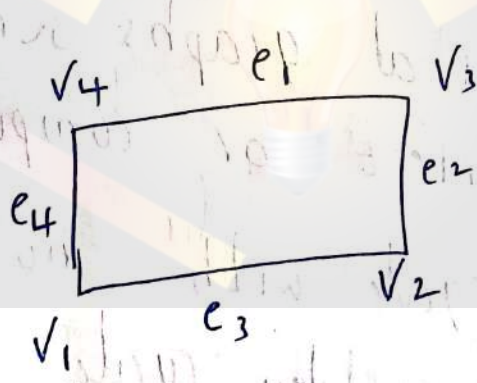
→ Line Graph:-

Let G be a graph with vertex $v \in V(G)$ and edge set $E(G)$. The line graph of the graph G is defined by $L(G)$ & defined with vertex set $V[L(G)] = E(G)$

$E(L(G)) = \{ ee' : e, e' \in E(G) \wedge e \text{ \& } e' \text{ are incident on same vertex.}$

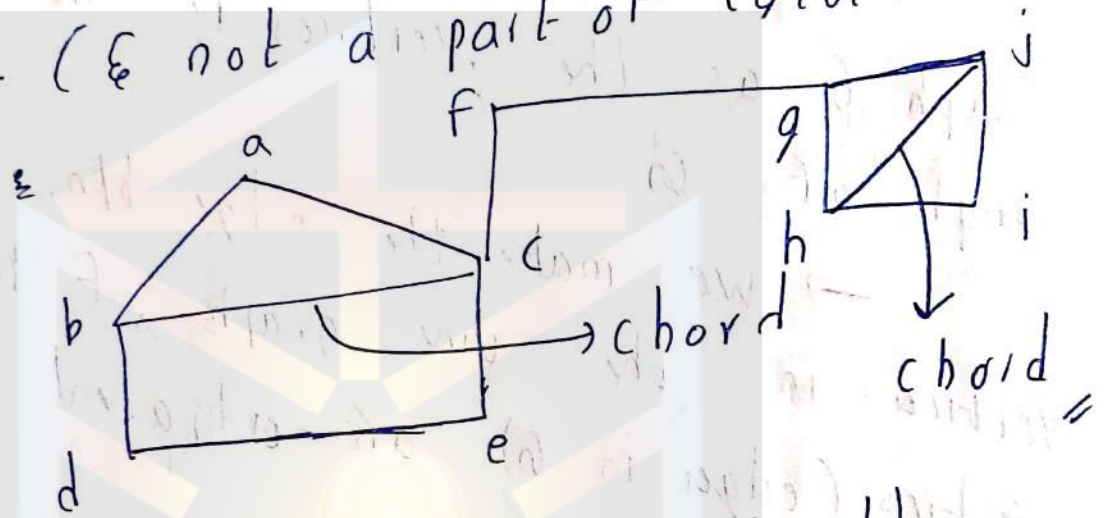
construction of the graph:

→ First we consider the edges of graph G as the vertices in the line graph of G .
 → We make the edge b/w two vertices in the line graph, if both (edges in G) are adjacent to each other.



→ Chordal graph:- is a simple graph in which every graph cycle of length 4 & greater has a chord.

→ chord is an edge connecting any 2 vertices of that cycle. (& not a part of cycle)

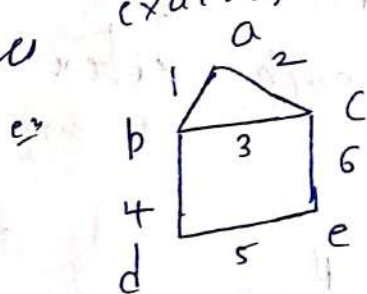


→ chordal graphs are the subset of perfect or complete graphs

→ A cycle with no chord is called a hole / chordless cycle

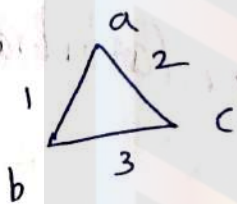
Eulerian/Euler graph:

→ euler path: is an open walk with all edges exactly once.



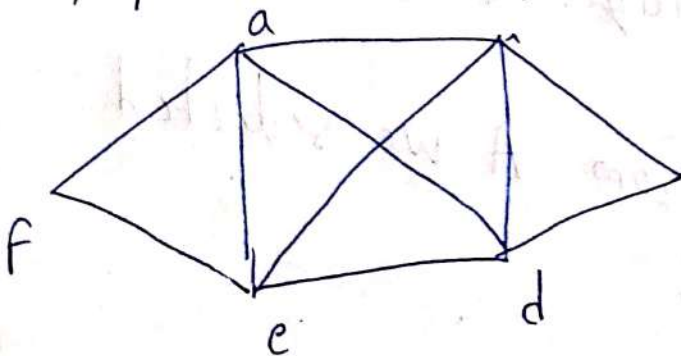
$b \rightarrow a \rightarrow c \rightarrow b \rightarrow d \rightarrow e \rightarrow c$

→ euler circuit: is a closed walk with all edges exactly once.



$a \rightarrow c \rightarrow b \rightarrow a$

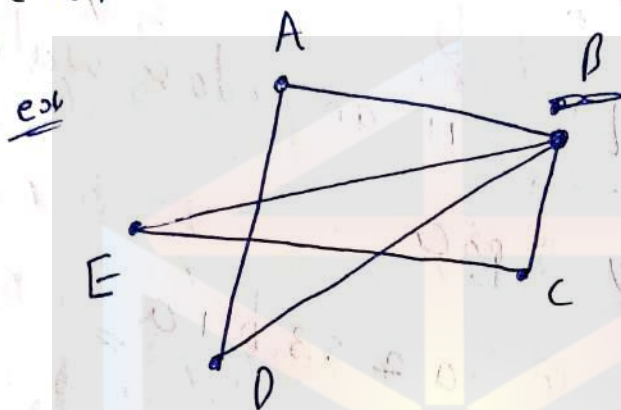
→ euler graph: A graph G which contains a euler cycle is called euler graph.



$a \rightarrow f \rightarrow e \rightarrow a$
 $a \rightarrow b \rightarrow d \rightarrow e \rightarrow b \rightarrow a$
 $a \leftarrow d \leftarrow c$

→ Fleury's algorithm ^{eulerian path}

→ To obtain eulerian circuit starting from any vertex in a given graph G in which each vertex degree is even.



Find a euler circuit

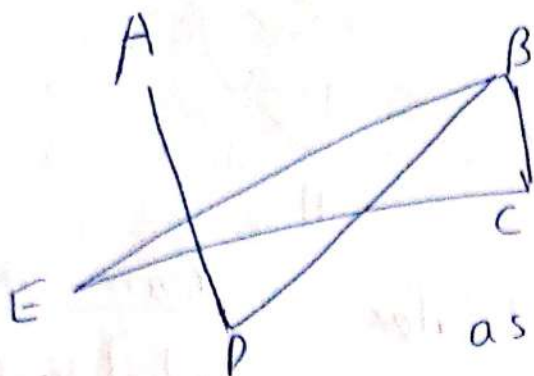
≡ Step 1: select a vertex (arbitrarily) any one

ass- A selected

Step 2: select an edge from V_A which is not a bridge & delete it from graph

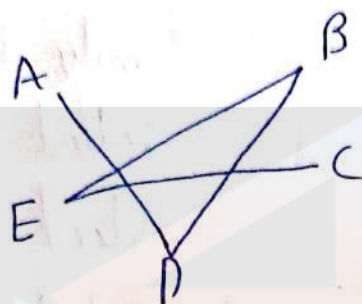
assume from A we selected
A B

⇒



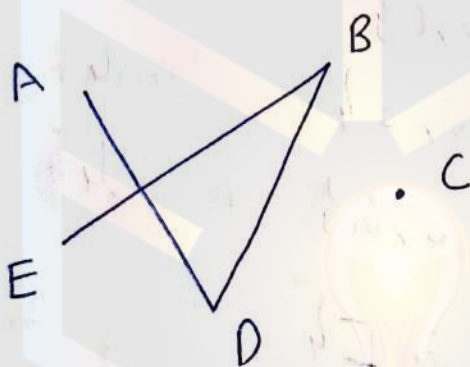
repeat step 2
at B now

assume selected BC
delete it



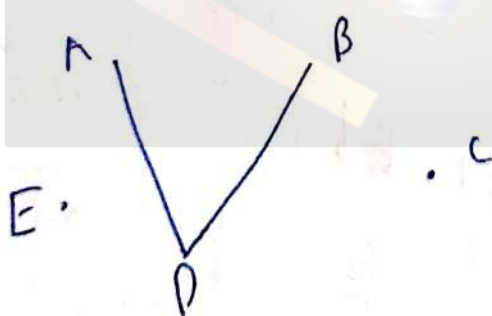
repeat step 2
at C now

assume selected CE
delete it



repeat step 2
at E now

assume selected EB
delete it



repeat step 2
at

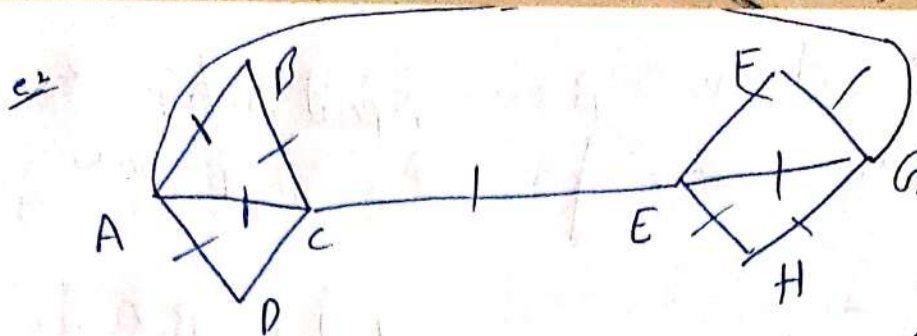
assume selected BD
delete it



Final AD

∴ ABCEBD A

∴ euler graph euler circuit



current-path

Next-Edge

Reason

$W_1 = A$

$\{A, B\}$

select any

$W_2 = A, B$

$\{B, C\}$

only $B \rightarrow C$

$W_3 = A, B, C$

$\{C, A\}$

select any

$W_4 = A, B, C, A$

$\{A, D\}$

select any

$W_5 = A, B, C, A, D$

$\{D, C\}$

select $D \rightarrow C$

$W_6 = A, B, C, A, D, C$

$\{C, E\}$

select $C \rightarrow E$

$W_7 = A, B, C, A, D, C$

$\{E, G\}$

select any

E

$\{G, F\}$

$W_8 = A, B, C, A, D, C,$

E, G

$\{F, E\}$

$W_9 = A, B, C, A, D, C,$

E, G F

$W_{10} = A, B, C, A, D, C,$

$\{E, H\}$

E, G, F, E

visited

$W_{11} = \{A, B, C, A, D, C\} \quad \{H, G\}$

E, G, F, E, H

$W_{12} = A, B, C, A, D, C, \quad \{G, A\}$

E, G, F, E, H, G

$W_{13} = A, B, C, A, D, C,$

E, G, F, E, H, G, A



Euler circuit

Hence given graph is Euler graph

→ Chinese post-man problem:-

→ Chinese mathematician Kuan proposed this problem

→ A postman receives the mail from post-office & delivers them & then returns to the post office.

→ so he wants to select shortest path / closed walk.

→ Finding closed walk that traverses each edge at least once.

Steps:- (calculate degree at each ~~edge~~ vertex)

i) if graph is eulerian (at each vertex the degree is ~~odd~~^{even}) return sum of all edges weight & follow to next step.

ii) We find all vertices with odd degree.

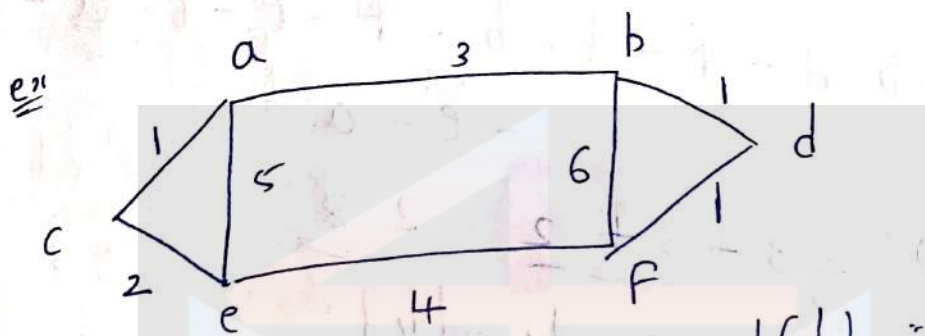
iii) List all possible pairing of odd vertices. For n odd vertices total no. of pairs

$$= (n-1) * (n-3) * \dots * 1$$

iv) Find each set of pairing v) Find the shortest path connecting them.

vi) Modify graph by adding all edges that have been found in step v)

vii) Weight of chueh postman tour
is sum of all edges in modified graph
viii) Print euler circuit of modified
graph i.e chueh postman tour.



$$\begin{aligned}
 d(a) &= 3 \quad \checkmark & d(d) &= 2 \\
 d(b) &= 3 \quad \checkmark & d(e) &= 3 \quad \checkmark \\
 d(c) &= 2 & d(f) &= 3 \quad \checkmark
 \end{aligned}$$

odd selected
 $\{a, b, e, f\}$

$$\{ \{a, b\} \{e, f\} \} = 3 + 4 = 7$$

$$\{ \{a, e\} \{b, f\} \} = 3 + 1 = 4 \quad \checkmark$$

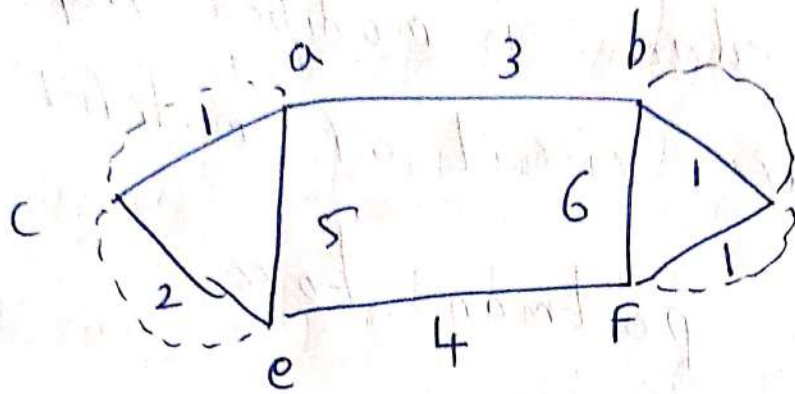
$$\{ \{a, f\} \{b, e\} \} = 1 + 4 = 5$$

$$(a, e) \& (b, f) \Rightarrow (b, d) \& (b, f)$$

$$\Downarrow$$

$$(a, c) \& (c, e)$$

updating graph



$a - b - d - f - d - b - f - e - c - a -$
 $c - e - a$

$$\Rightarrow \underbrace{2}_{\text{normal}} \underbrace{3 + 3 + 2}_{\text{newly added}} = \underline{\underline{28}}$$