Code No. 11589/CBCS

FACULTY OF ENGINEERING

B.E. VI – Semester (CBCS) (CSE) (Main) Examination, April/May 2019

Subject: Design & Analysis of Algorithms

Time: 3 Hours **Max. Marks:70**

PART – A (2 x 10 = 20 Marks)

1. Explain asymptotic notations.
2. What is Knapsack Problem?
3. Why Multi stage graphs are needed?
4. Write Weighing Union algorithm?
5. Solve the recurrence relation

   T(n) = T(1) n = 1

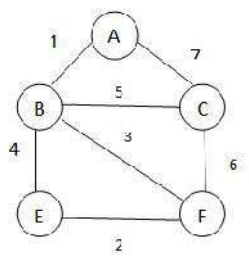
   = a T(n/b)+f(n) n > 1

   for a, b=2, T(1) = 2 and f(n) = n
6. Define E-Node, live node and dead node.
7. List out the NP-Hard code generation problems.
8. Define DFS with example.
9. List the differences between divide and conquer and Greedy method.
10. State purging Rule and list out its applications.

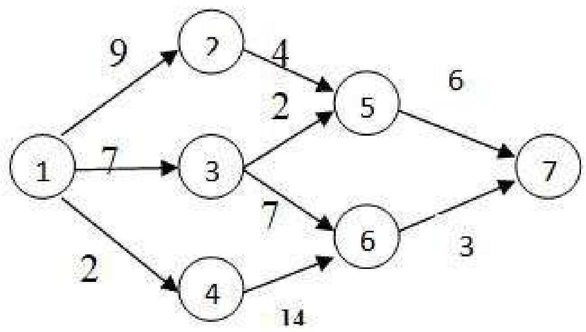
Part – B (5 x 10 = 50 Marks)

11. Explain Heap sort with algorithm and construct Max heap tree for the set of Numbers 15,8,20,12,9,1,34,4 (10M)

12. a) What is single source shortest path problem. Explain with an example. (5M)
    b) Define Spanning tree and explain Kruskal's algorithm for finding minimum cost spanning tree for the given graph.



13. What is multistage graph? Find the shortest path in the graph given bellow using Dynamic Programming. (10M)



14. Solve the knapsack problem using backtracking for the following problem: P={11,21,31,33,43,53,55,65}, w={1,11,21,23,33,43,45,55}, m=110, n=8. (10M)

15. Draw the portion of the state space generated by LCBB for the knapsack Instance. n=4,(p1,p2,p3,p4)=(10,10,12,18),(w1,w2,w3,w4)=(2,4,6,9) and m=15. (10M)

16. Explain job sequencing with deadlines algorithm and also find the solution for the instance n=7,(p1,p2,…p7)=(3,5,20,18,1,6,30) and (D1,D2,..,D7)= (1,3,4,3,2,1,2) (10M)

17. For the identifier set (a1,a2,a3,a4)=(count, float, if, while) with P(1)=1/20, P(2)=1/5, P(3)=1/10,P(4)=1/20,q(0)=1/5, q(1)=1/10, q(2)=1/5,q(3)=1/20, and q(4)=1/20 construct the OBST. (10M)

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Code No: 255

FACULTY OF ENGINEERING

BE 3/4 (CSE) II-Semester (Main & Backlog) Examination, May / June 2018

Subject: Design & Analysis of Algorithms

Time: 3 Hours **Max. Marks: 75**

Note: Answer all questions of Part-A & Answer any FIVE Questions from Part-B.

PART-A (25 Marks)

1. In the algorithm given, check whether you can reduce the execution time by any means; [2]

```
Fun (n)
if n=1 then
return 1;
else
return fun (n-1)+fun(n-1)
```

2. An array has 'n' nodes, filled with set {0,1,.......} There are no duplicates. Design an O(n) worst case algorithm to find which element from the above set is missing [3]
3. What is the drawback of greedy algorithm? [2]
4. Show the intermediate steps when the number {123, 23, 1, 43, 54, 36, 75, 34} are sorted using merge sort. [3]
5. Why multistage graphs are needed? [3]
6. What is Knapsack Problem? [2]
7. Define chromatic number of a graph. [2]
8. Define a strongly connected digraph and give the minimum in-degree of all nodes in the graph [3]
9. State node covering decision problem [2]
10. What is NP completeness? [3]

PART-B (50 Marks)

11. (a) Define asymptotic notation with examples with time and space complexity estimation [4]
    (b) Write Binary search algorithm and analyze its time complexity [6]

..2

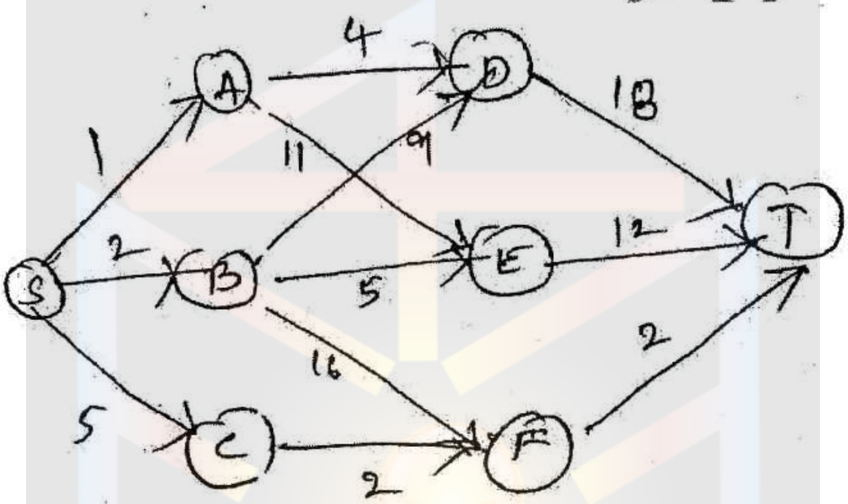
12. (a) Using Divide and conquer approach find max and min in a set if 'n' elements. Also find the recurrence relation for the number of elements compared [5]

(b) Find an optimal solution to the Kanpsack instance n=7, m=15

$(P_1, P_2, P_3 \ldots\ldots\ldots P_7) = (10, 5, 15, 7, 6, 18, 3)$

$(W_1, W_2 \ldots\ldots\ldots\ldots W_7) = (2, 3, 5, 7, 1, 4, 1)$ [5]

13. (a) Using dynamic programming solve the following graph using backward approach [10]



14. Using backtracking, find optimal solution to Knapsack problem for instance n=7, m=110 $(P_1, P_2 \ldots\ldots P_7) = (11, 21, 32, 33, 43, 53, 55, 65)$ and $(W_1 \ldots\ldots W_7) = (1, 11, 21, 33, 43, 53, 55, 65)$ [10]

15. (a) What is NP-Hard code generation problem [5]

(b) Give details about logic programming in perspective problem. [5]

16. What is Hamilkonian problem? Explain with example using backtracking [10]

17. Write a short notes on

(a) Lower Bound Theory [5]

(b) Heapsort working procedure [5]

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Code No. 136 / O

FACULTY OF ENGINEERING

B.E. 3/4 (CSE) I – Semester (Old) Examination, December 2017

Subject: Design and Analysis of Algorithms

Time: 3 Hours **Max.Marks: 75**

Note: Answer all questions from Part A and any five questions from Part B.

PART – A (25 Marks)

1 Solve the recurrence relation T(n) = 9T(n/3)+n. 3

2 List out collision resolution techniques in hashing. 2

3 Define optimal storage on tapes. 3

4 Write control abstraction for greedy method. 2

5 List out the properties of Bi connected graphs. 3

6 Define principle of optimality. 2

7 What are explicit and implicit constraints in backtracking algorithm? 3

8 What is a Hamiltonian cycle? Give an example. 2

9 State Cook's theorem. 2

10 Differentiate FIFO and LC branch and bound techniques. 3

PART – B (50 Marks)

11 a) What is asymptotic complexity of an algorithm? Give an example of an algorithm whose complexity is O (nlogn). 4

UNION (1,2,2)
UNION (2,3,3)
:
:
UNION (n-1, n, n)
FIND (1)
FIND (2)
FIND (n)

i) Write the tree after (n-1) UNION operations. 3
ii) Compute the cost of n FIND instructions. 3

12 a) Write an algorithm to sort N numbers in descending order using Quick sort and analyze time complexity in best, average, worst cases. 5

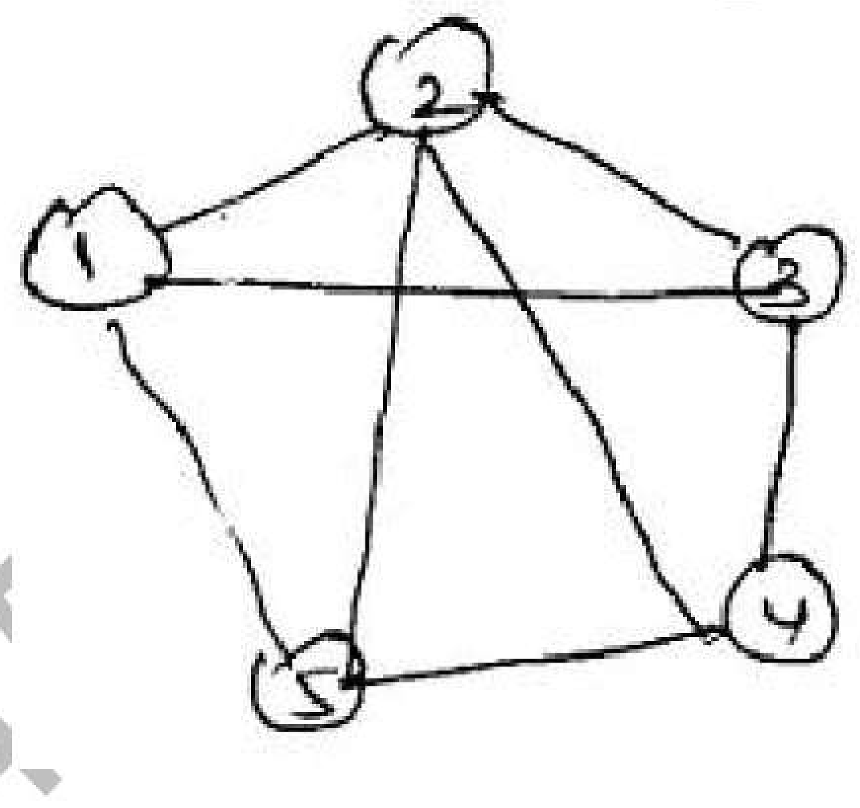
b) Consider the following instance of knapsack problem n=7, m=21
(P1, P2, P3, P4, P5, P6, P7) = (10, 5, 15, 7, 6, 18, 3) and
(W1, W2, W3, W4, W5, W6, W7) = 2, 3, 5, 7, 1, 4, l1,) solve by using Greedy approach. 5

13 a) How to find articulation point and Bi-connected components from given graph? Explain with an example. 5

b) Design a three stage system with devices D1, D2, D3. The costs are $30, $15, $20 respectively. The cost of the system is to be no more than $105. The reliability of each device type is 0.9, 0.8, 0.5. 5

14 a) Present an algorithm how the techniques of back tracking can be applied to solve the 8 Queens problem. 5

b) Define chromatic number. Draw the state space tree for the following graph with n=5, m=3. 5



15 a) Write non deterministic algorithm for searching. 4

b) Explain in brief, NP Hard and NP Complete problems. 6

16 a) Construct OBST for the following data n=4, (a1, a2, a3, a4) = (do, if, int, while) and P(1:4) = (3, 3, 1, 1) and Q(0:4) = (2, 3, 1, 1, 1). 6

b) Write an algorithm to find maxmin of a given list of elements. 4

17 Write short notes on any <u>two</u> of the following: 10
a) Priori and posteriori analysis
b) Satisfiability problem
c) Big oh and Theta notation.

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Code No. 225 / N

FACULTY OF ENGINEERING

B.E. 3/4 (CSE) II – Semester (New) (Suppl.) Examination, May 2017

Subject: Design & Analysis of Algorithms

Time: 3 Hours **Max.Marks: 75**

Note: Answer all questions from Part A and any five questions from Part B.

PART – A (25 Marks)

1	Why an algorithm analysis required?	2
2	State about 'union' and 'find' operations.	2
3	Define feasible solution w.r.t greedy knapsack problem.	3
4	Compare Quick sort Vs Merge sort.	3
5	What is a Bi-connected components.	2
6	State 4 queens problem.	3
7	Write an algorithm for finding minimum element in a list of elements.	3
8	Write about Least Count Search.	2
9	Define Chromatic Number.	2
10	Discuss Clique Decision Problem.	3

PART – B (5x10 = 50 Marks)

11 Explain step by step heap sort with example and write an algorithm for heap sort. 10

12 a) Explain about Huffman codes. 5

b) Find feasible solution for given jobs with deadlines,
n=7, (p1, p2 … p7)=(3,5,20,18,1,6,30), 9d1, d2 … d7) = (1,3,4,3,2,1,2). 5

13 Explain about all pairs shortest path problem with an example and write algorithm. 10

14 Consider Travelling sales person instance 10

$$\begin{pmatrix} \alpha & 7 & 3 & 12 & 8 \\ 3 & \alpha & 6 & 14 & 9 \\ 5 & 8 & \alpha & 6 & 18 \\ 9 & 3 & 5 & \alpha & 11 \\ 18 & 14 & 9 & 8 & \alpha \end{pmatrix}$$

Obtain the state space tree that is generated by LCBB.

15 Explain about NP-Hard scheduling problems. 10

16 Construct Optimal Binary Search Tree and Compute w(i, j), r(i, j), c(i, j), 0<=i,j, j<=4, for the identifier set (a1, a2, a3, a4) = (count, float, ifr, while) with p(1)=1/20, p(2)=1/5, p(3)=1/10, p(4)=1/20, q(0)=1/5, q(1)=1/10, q(2)=1/5, q(3)=1/20, q(4)=1/20. 10

17 Explain about DFS with example. Write algorithm and its time complexity. 10

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Code No. 3481 / N

FACULTY OF ENGINEERING

B.E. 3/4 (CSE) II – Semester (New) (Main) Examination, May/June 2017

Subject: Design & Analysis of Algorithms

Time: 3 Hours **Max.Marks: 75**

Note: Answer all questions from Part A and any five questions from Part B.

PART – A (25 Marks)

1	Define Hashing.	2
2	Give DFS example	2
3	Write about optimal storage on tapes	3
4	Write an algorithm for finding maximum element in a list of elements.	3
5	Why Multistage Graphs are needed?	2
6	What is Least Count Search?	3
7	How graph coloring is applied to real world.	3
8	Define the term Branch and Bound.	2
9	Discuss Node cover decision problem.	3
10	Define NP-Hard problem.	2

PART – B (5x10 = 50 Marks)

11	Write merge sort algorithm and derive its time complexity for best and worst cases.	10
12	a) Explain about minimum spanning trees with examples.	5
	b) What is single source shortest path problem? Explain with example.	5
13	a) Explain about 0/1 Knapsack problem using dynamic programming.	4
	b) Draw the state space tree generated by LCBB for n=5, (p1, p2, p3, p4, p5) = (10, 15, 6, 8, 4), (w1, w2, w3, w4, w5) = (4, 6, 3, 4, 2), m=12.	6
14	Explain about Hamilton cycles and write an algorithm for Hamilton cycle.	10
15	Explain about cook's theorem.	10
16	a) Explain about different Asymptotic Notations.	5
	b) Find the time complexity of T(n), $T(n) = T(n-1)+2$ if $n>0$; $= 1$ if $n=0$	5
17	Solve 8 queens problem using backtracking.	10

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Code No. 3154 / O

FACULTY OF ENGINEERING

B.E. 3/4 (CSE) I - Semester (Old) Examination, November / December 2016

Subject : Design and Analysis of Algorithms

Time : 3 Hours **Max. Marks: 75**

Note: Answer all questions from Part-A and answer any five questions from Part-B.

PART – A (25 Marks)

1 Write the properties of Big-oh Notation. (3)

2 Show the intermediate steps when numbers 123, 23, ', 43, 54, 36, 75, 34 are sorted using merge sort. (3)

3 What do you mean by divide and conquer strategy? (3)

4 What is resultant max-heap for numbers ? "32, 15, 20, 30, 12, 25, 16" (3)

5 Define optimal Binary search tree. (2)

6 What is chromatic number ? (3)

7 Draw a graph with no Hamiltonian cycle. (2)

8 What is Cook's theorem? (2)

9 State the principle of Back tracking method. (2)

10 Differentiate P and NP class of problems. (2)

PART – B (50 Marks)

11 (a) Do the performance analysis for selection sort for the Best, worst and average cases? (5)

(b) Use linear probing, a hash table with bF13 buckets and has function f(K) = K % b. Insert pairs whose keys are 7, 42, 25, 70, 14, 38, 8, 21, 34, 11. (5)

(i) Show hash table following each insert

(ii) What is max and average number of buckets examined in successful search

12 (a) Implement Quick sort and derive the worst case time complexity. (6)

(b) Using O/I knapsack technique, find optimal solution for the given problem capacity = 30 (4)

Time	A	B	C
Price	50	140	60
Size	5	20	10

13 (a) Explain all-pairs shortest path Algorithm. (5)

(b) Use Dynamic programming to obtain a solution for Travelling salesman problem that runs in $O(2^n)$ time. (5)

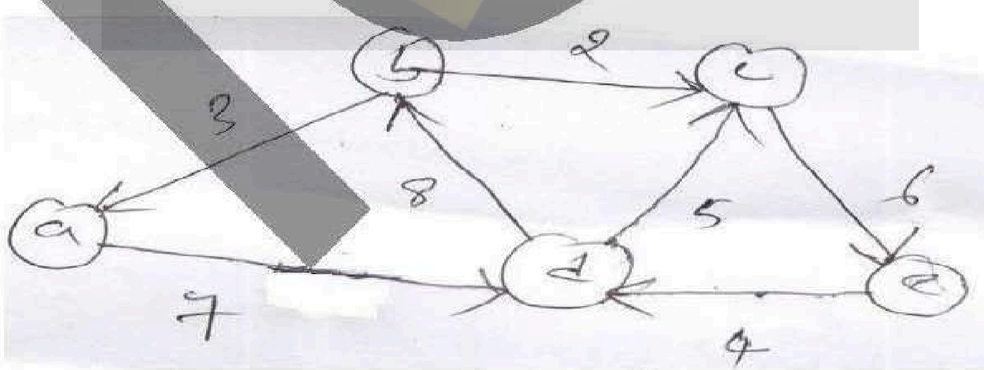
14 (a) How backtracking works on 8-Queens problem with example? (6)

(b) Differentiae explicit and implicit constraints of a Backtracking solutions. (4)

15 (a) Write short notes on NP-hard and NP-complete problems (5)

(b) Explain Node covering decision problem. (5)

16 Implement Depth first search and find all nodes reachable from 'a' in the below graph. (10)



17 (a) Generate minimum spanning tree using Prims algorithm write and analyse the algorithm used. (6)

(b) How is graph coloring done using Backtracking method? Explain the algorithm used to find Next color. (4)

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Code No. 9116 / S

FACULTY OF ENGINEERING

B.E. 3/4 (CSE) I-Semester (Suppl.) Examination, June / July 2015

Subject : Design and Analysis of Algorithms

Time : 3 Hours Max. Marks: 75

Note: Answer all questions from Part - A and answer any five questions from Part-B.

PART – A (25 Marks)

1 Differentiate between profiling and debugging. (3)

2 Write an algorithm to find factorial of a number 'n'. (2)

3 Write short notes on Selection problem. (3)

4 Differentiate Greedy method with divide-conquer method. (2)

5 How many number of articulation points does a biconnected graph have? Justify. (2)

6 What is an optimal binary search Tree? (3)

7 What is Hamiltonian cycle? (3)

8 Draw a comparison tree for sorting three number x, y, z. (3)

9 State halting problem. (2)

10 When do you say that two problems X, Y are polynomially equivalent? (2)

PART – B (50 Marks)

11 (a) Give an array of n elements (may contain duplicate elements), write an algorithm to remove all duplicates from the array. (6)
(b) Write note on UNION, FIND operations in set theory. (4)

12 (a) Does either prim's algorithm or kruskal's algorithm work if there are negative edge weights. (3)
(b) Describe knapsack problem with example. (7)

13 Construct an optimal binary search tree for the following data, (10)
n=4 (a_1, a_2, a_3, a_4)= (do, if, int, while), p(1:4)=(3, 3, 1, 1) and q(0:4)=(2, 3, 1, 1, 1).

14 Extract the fundamental idea behind dynamic programming inorder to solve computer related problems. (10)

15 Explain in detail any 3 problems that can be solved using backtracking approach. (10)

16 (a) What is recurrence relation? Explain. (5)
(b) Write notes on Hashing. (5)

17 Write notes on any **three** of the following: (10)
(a) Satisfiability problem
(b) Reliability Design
(c) NP Hard graph problems
(d) Node covering problem

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Code No. 9116

FACULTY OF ENGINEERING

B.E. 3/4 (CSE) I-Semester (Main) Examination, December/January 2014-15

Subject : Design and Analysis of Algorithms

Time : 3 Hours Max. Marks: 75

Note: Answer all questions of Part - A and answer any five questions from Part - B.

PART – A (25 Marks)

1 What do you mean by performance analysis of an algorithm? Explain. (3)
2 Explain the significance of input size of a problem. (2)
3 Give the control abstraction for Greedy method. (2)
4 What is the time required to compute a minimum weight spanning tree for any given weighted graph with E edges and V vertices? (2)
5 Differentiate greedy method and dynamic programming. (3)
6 List out the properties of biconnected graphs. (2)
7 Define satisfiability problem. (3)
8 List out the various NP-Hard graph problems. (3)
9 Define the greedy Knapsack problem. (3)
10 What is the basic requirement of a planar graph? (2)

PART – B (50 Marks)

11 (a) Derive the time complexity of Quick Sort algorithm. (7)
(b) What is the significance of a good hash function? (3)

12 Explain job sequencing with deadline algorithm and also find the solution for the instance n = 7,
(p1, p2, ..., p7) = (3, 5, 20, 18, 1, 6, 30) and
(D1, D2, ...D7) = (1, 3, 4, 3, 2, 1, 2)

13 (a) Explain principle of optimality. (5)
(b) Explain the forward and backward approaches of problem solving in dynamic programming. (5)

14 (a) Write an algorithm for 8-Queen's problem using back tracking. (6)
(b) Explain the technique of backtracking. (4)

15 (a) Differentiate between NP-complete and NP-Hard. (4)
(b) Write a non-deterministic algorithm for sorting. (6)

16 Explain in detail any two problems that can be solved using divide and conquer strategy (10)

17 Briefly explain any **two** of the following: (5 + 5)
(a) Optimal Binary Search Tree (b) Graph Coloring
(c) Big-O notation

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B.E. 3/4 (CSE) I – Semester (Main) Examination, November 2013

FACULTY OF ENGINEERING

Code No. 6112

Subject : Design and Analysis of Algorithms

Time : 3 hours

Max. Marks : 75

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1. Write the control abstraction for Divide and Conquer technique. (2)
2. Solve : $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$. (3)
3. Explain set representation and write algorithm for 'FIND'. (3)
4. What is Hamiltonian cycle? How is it different from the tour of travelling salesperson problem? (2)
5. Explain optimal merge pattern with an example. (3)
6. Differentiate between Greedy and Dynamic programming approaches. (2)
7. Explain graph coloring problem. (3)
8. What is meant by lower bound theory. (2)
9. Solve the fractional knapsack problem by considering the instance : Weights are, W : {1, 3, 5, 6, 7}, Profits, P : {3, 9, 7, 11, 18}. The knapsack capacity is 15. (3)
10. Explain what are explicit and implicit constraints of 8-Queens problem. (2)

PART – B (50 Marks)

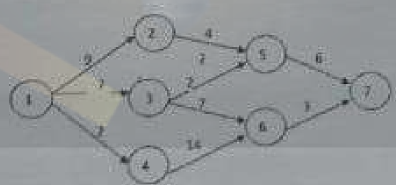
11.a) Draw the 11 entry hash table for hashing the keys 12, 44, 13, 88, 23, 94, 11, 39, 20 using the function (2i+5) mod 11, using linear probing. (5)

b) Write an algorithm to form a heap using 'Heapify' and discuss about its time complexity. (5)

12.a) Define spanning tree and explain Prim's algorithm for finding minimum spanning tree of the graph (5) using the graph given below (5)

b) Write an algorithm for merge sort and write the time complexities. (5)

13. What is Multistage graph? Find the shortest path in the graph given below dynamic programming. (10)



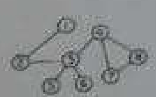
14.a) Explain solution of graph coloring problem using backtracking. (5)

b) What is Branch and bound strategy? Explain. (5)

15.a) State Cook's theorem. Explain its significance in NP-complete theory. (5)

b) Discuss NP-Hard code generation problems. (5)

16. Define biconnected component of a graph. Identify articulation point and draw biconnercted components of. (10)



17. Write short notes on :
a) Reliability Design (5)
b) Job scheduling with deadlines using Greedy approach (5)

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Code No. 2354 / N

FACULTY OF ENGINEERING

B.E. 3/4 (CSE) I-Semester (New)(Main) Examination, November / December 2012

Subject : Design and Analysis of Algorithms

Time : 3 Hours

Max. Marks: 75

Note: Answer all questions of Part - A and answer any five questions from Part-B.

PART – A (25 Marks)

1. Show the following:
   (a) $10n^2 + 9 \neq O(n)$ (b) $n^3 + 10n^2 = \Theta(n^3)$ (3)
2. Solve the recurrence: $T(n) = 6T(n/3)+n^2 \log n$ (3)
3. Write the control abstraction for Greedy approach. (2)
4. What is principle of Optimality? (2)
5. Explain traveling salesperson problem. (3)
6. What is meant by Satisfiability? (2)
7. What is Hamiltonian cycle? (2)
8. What is DFS and list its applications? (2)
9. Define the properties of LC-Search. (3)
10. Find an optimal binary merge pattern for files whose lengths number of records are 2, 5, 7, 9, 12, 13, 15 (3)

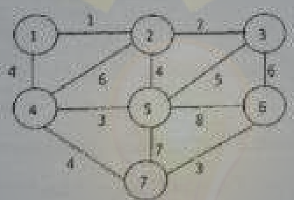
PART – B (5x10=50 Marks)

11.(a) What are the collision resolution policies in hashing? Write an algorithm for hashing with linear probing. (5)

(b) Sort the following numbers 3, 16, 12, 14, 11, 15 using Heap sort. Show the step by step procedure. (5)

12.(a) Write a recursive algorithm for finding both the minimum and maximum elements in an array A of n elements. What is the running time? (5)

(b) Define spanning tree and explain Kruskal's algorithm for finding Minimum Spanning Tree of the graph using the graph given below and write its time complexity. (5)



13. Write recurrence relations for solving OBST using dynamic programming and construct the tree for given data: (10)
    $n=4$, (a_1, a_2, a_3, a_4) = (end, goto, print, stop)
    $p(1:4)$=(1/20, 1/5, 1/10, 1/20) $q(0:4)$=(1/5, 1/10, 1/5, 1/20, 1/20)
    Where Ps are probability of successful search and qs are probability of unsuccessful search.

14.(a) Write an algorithm for n Queens using backtracking approach. (7)

(b) Explain FIFO Branch and Bound. (3)

15.(a) Explain what are NP-Hard and NP-Complete problems. (5)

(b) Explain node cover decision problem. (5)

16.(a) Write an algorithm for in place partitioning of elements, taking first element of an array as pivot element. (5)

(b) Find an optimal solution to 0/1 knapsack when (5)
(w_1, w_2, w_3, w_4)=(10, 15, 6, 9)
(p_1, p_2, p_3, p_4) = (2, 5, 8, 1)
Knapsack capacity = 25 where Wi's are weights and Pi's are profits.

17. Write a short note on : (5+5)
    (a) Optimal storage on tapes (b) Multistage graphs

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FACULTY OF ENGINEERING

B.E. 3/4 (CSE) I – Semester (Supplementary) Examination, July 2014 Code No. 6112 / S

Subject : Design and Analysis of Algorithms

Time : 3 hours Max. Marks : 75

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1 Present an algorithm that searches an unsorted array a[1 : n] for the element 'x'. If 'x' occurs, then returns a position in the array, else return zero? 3
2 State the weighting, collapsing rules in sets. 2
3 Briefly differentiate quick sort and merge sort. 2
4 Define the terms Feasible solution, optimal solution and objective function. 3
5 State the purging rule and list out its applications. 2
6 Draw a five-stage graph. 3
7 What is the objective of m-colorability optimization problem? 2
8 Differentiate FIFO, LIFO branch-and-bound. 3
9 Define the terms cliques, node cover. 2
10 What are NP-Hard code generation problems? 3

PART – B (5 x 10 = 50 Marks)

11 a) Explain back tracking. Give the various applications of backtracking.
b) Solve the 8-Queen's problem using backtracking.

12 Briefly explain the terms
a) Non-determinstic algorithms b) Satisfiability problem c) Reducibility

13 Solve the 0/1 knapsack instance where
$n = 3$, $(w_1, w_2, w_3) = (2, 3, 4)$, $(p_1, p_2, p_3) = (1, 2, 5)$ and 6 using Dynamic programming.

14 a) Present an algorithm 'select' that finds the k^{th} smallest element in an array a[1:n].
b) Briefly explain spanning trees and their applications.

15 Present the heap sort algorithm. Explain and analyze its time complexity.

16 What are comparision trees? Explain their applications in searching and sorting problems?

17 Solve the all-pairs shortest path problem for a diagraph with the following weight matrix?

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 11 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & \infty & 0 \end{array}$$

******

Dec / Jan 2014-15 , April / May 2019

12) deadline algorithm ↳ 16)

- Given

$(P_1, P_2, P_3, \dots, P_7)$ = 3, 5, 20, 18, 1, 6, 30

$(D_1, D_2, D_3, \dots, D_7)$ - 1, 3, 4, 3, 2, 1, 2

	1	2	3	4	5	6	7
P	3	5	20	18	1	6	30
D	1	3	4	3	2	1	2

- Possible deadline → 1, 2, 3 & 4

all possible

[| | |]
1 2 3 4

at 1: 1, 2, 3, 4, 5, 6, 7

at 2: only item 2, 3, 4, 5, 7

at 3: only items 2 or 4 or 3

at 4: only item 3 has chance.

after removing impossible things like if 3 selected at 4th possition it cannot be selected at 3 etc...

removing them

[| | |]
1 2 3 4

at 1: 1, 2, 4, 5, 6, 7

at 2: 2, [illegible], 4, 5, 7

at 3: 2 or 4

at 4: only 3

all combination

Feasible solution	Sequence	(Sum of) Profit ΣP_i
{1, 2, 4, 3}	1243, 1423	46
{1, 5, 2, 3}	1523	29
{1, 5, 4, 3}	1543	42
{1, 7, 2, 3}	1723	58
{1, 7, 4, 3}	1743	71
{2, 5, 4 3}	2543, 4523, 5243, 5423	44
{2, 7, 4 3}	2743, 4723, 7432, 7243	73
{5, 7, 2, 3}	5723, 7523	56
{5, 7, 4, 3}	5743, 7543	49
{6, 2, 4 3}	6243, 6423	32
{6, 5, 2, 3}	6523	~~45~~ 32
{6, 5, 4, 3}	6543	45
{6, 7, 2, 3}	6723	74
{6, 7, 4 3}	67243	

6	7	4	3
1	2	3	4

→ 74 ← max profit

→ June/ July 2015

13) Optimal binary tree.
in SW unit 3

→ Dec 2017

16) a) same problem

→ April/May 2019, May/June 2018

14 14

$+ w(i,j)$

$w(i,j) = w(i, j-1) + p(j) + q(j)$

$r(i,j) = k$

ex:- $n = 4$; $(a_1, a_2, a_3, a_4) = $ (do, it, int, while)

$p(1,2,3,4) = (3,3,1,1)$ $q(0,1,2,3,4) = (2,3,1,1,1)$

as $n = 4$

we need to find till T_{04}

i) $j - i = 0$

$\Rightarrow T_{00}, T_{11}, T_{22}, T_{33}, T_{44}$.

T_4 ?

i) $j - i = 0$
ii) $j - i = 1$
$j - i = 2$
3
4

Note

$$\left.\begin{array}{l} C(i, i) = 0 \\ r(i, i) = 0 \\ W(i, i) = q(i) \end{array}\right\}$$

$$T_{00} \Rightarrow \left.\begin{array}{l} C(0,0) = 0 \\ r(0,0) = 0 \\ W(0,0) = 2 \end{array}\right\}$$

$$T_{11} \Rightarrow \left.\begin{array}{l} C(1,1) = 0 \\ r(1,1) = 0 \\ W(1,1) = q(1) = 3 \end{array}\right\}$$

$$T_{22} \Rightarrow \left.\begin{array}{l} C(2,2) = 0 \\ r(2,2) = 0 \\ W(2,2) = q(2) = 1 \end{array}\right\}$$

$$T_{33} \Rightarrow \left.\begin{array}{l} C(3,3) = 0 \\ r(3,3) = 0 \\ W(3,3) = q(3) = 1 \end{array}\right\}$$

$$T_{44} \Rightarrow \left.\begin{array}{l} C(4,4) = 0 \\ r(4,4) = 0 \\ W(4,4) = q(4) = 1 \end{array}\right\}$$

ii) $j - i = 1$

$\rightarrow$ ~~$T_{10}, T_{21}, T_{32}, T_{43}$~~

T_{01}, T_{12}, T_{23}, T_{34}

Note:

$w(i,j) = w(i, j-1) + p(j) + q(j)$

$c(i,j) = \min_{i < k \leq j} \{ c(i, k-1) + c(k,j) \} + w(i,j)$

$r(i,j) = k$

$T_{01} \Rightarrow w(0,1) = w(0,0) + p(1) + q(1)$

$= 2 + 3 + 3 = 8$

$c(0,1) = \min_{0 < k \leq 1} \{ c(0,0) + c(1,1) \} + w(0,1)$

$= 0 + 0 + 8 = 8$

$r(0,1) = k = 1$

$T_{12} \Rightarrow w(1,2) = w(1,1) + p(2) + q(2)$

$= 3 + 3 + 1 = 7$

$c(1,2) = \min_{1 < k \leq 2} \{ c(1,1) + c(2,2) \} + w(1,2)$

$= 0 + 0 + 7 = 7$

$r(1,2) = k = 2$

$T_{23} \Rightarrow w(2,3) = w(2,2) + P(3) + q(3)$

$= 1 + 1 + 1 = 3$

$c(2,3) = \min_{2 < k \leq 3} \{ c(2,2) + c(3,3) \} + w(2,3)$

$= 0 + 0 + 3 = 3$

$r(2,3) = k = 3$

$T_{34} \Rightarrow w(3,4) = w(3,3) + P(4) + q(4)$

$= 1 + 1 + 1 = 3$

$c(3,4) = \min_{3 < k \leq 4} \{ c(3,3) + c(4,4) + w(3,4) \}$

$= 0 + 0 + 3$

$r(3,4) = k = 4$

iii) $j - i = 2$

T_{02}, T_{13}, T_{24}

$T_{02} \Rightarrow w(0,2) = w(0,1) + P(2) + q(2)$

$= 8 + 3 + 1 = 12$

$c(0,2) = \min_{\substack{0 < k \leq 2 \\ k = 1, 2}} \left\{ \begin{array}{l} c(0,0) + c(1,2) \\ c(0,1) + c(2,2) \end{array} \right\} + w(0,2)$

$c(0,0) + c(1,2)$

$c(0,1) + c(2,2)$

$c(i, k-1) + c(k, j)$

$$= \begin{Bmatrix} 0 + 7 \\ 8 + 0 \end{Bmatrix} + 12 = \underline{19}$$

$r(0,2) = k = \underline{1}$

$T_{13} \Rightarrow W(1,3) = W(1,2) + P(3) + Q(3)$

$= 7 + 1 + 1 = 9$

$$C(1,3) = \min_{\substack{1 < k \le 3 \\ k = 2,3}} \begin{Bmatrix} C(1,1) + C(2,3) \\ C(1,2) + C(3,3) \end{Bmatrix} + W(1,3)$$

$$= \min \begin{Bmatrix} 0 + 3 \\ 7 + 0 \end{Bmatrix} + 9 = \underline{12}$$

$r(1,3) = k = 2$

$T_{24} \Rightarrow W(2,4) = W(2,3) + P(4) + Q(4)$

$= 3 + 1 + 1 = 5$

$$C(2,4) = \min_{\substack{2 < k \le 4 \\ k = 3,4}} \begin{Bmatrix} C(2,2) + C(3,4) \\ C(2,3) + C(4,4) \end{Bmatrix} + W(2,4)$$

$$= \begin{Bmatrix} 0 + 3 \\ 3 + 0 \end{Bmatrix} + 5 = 8$$

$r(2,4) = k = 3$ (3 or 4 ~~both~~ any 1 select)

iv) $j - i = 3$

T_{03} , T_{14}

$T_{03} \Rightarrow T = W(0,2) + P(3) + Q(3)$ (above: $w(0,3)$)

$= 12 + 1 + 1 = 14$

$$C(0,3) = \min_{\substack{0 < k \le 3 \\ k = 1,2,3}} \begin{Bmatrix} C(0,0) + C(1,3) \\ C(0,1) + C(2,3) \\ C(0,2) + C(3,3) \end{Bmatrix} + W(0,3)$$

C (0 0 | 1 3; 0 1 | 2 3; 0 2 | 3 3)

i, k-1, k, j

$$= \begin{Bmatrix} 0 + 12 \\ 8 + 3 \\ 19 + 0 \end{Bmatrix} + 14 = 25$$

$r(0,3) = k = 2$

$T_{14} \Rightarrow W(1,4) = W(1,3) + P(4) + Q(4)$

$= 9 + 1 + 1 = 11$

$$C(1,4) = \min_{1 < k \le 4} \begin{Bmatrix} C(1,1) + C(2,4) \\ C(1,2) + C(3,4) \\ C(1,3) + C(4,4) \end{Bmatrix} + W(1,4)$$

$$= \min \begin{Bmatrix} 0 + 8 \\ 7 + 3 \\ 12 + 0 \end{Bmatrix} + 11 = 19$$

$r(1,4) = k = 2$

v) $j - i = 4$

T_{04}

$T_{04} \Rightarrow w(0,4) = w(0,3) + p(4) + q(4)$

$= 14 + 1 + 1 = 16$

$$c(0,4) = \min_{\substack{0 < k \le 4 \\ k = 1,2,3,4}} \begin{Bmatrix} c(0,0) + c(1,4) \\ c(0,1) + c(2,4) \\ c(0,2) + c(3,4) \\ c(0,3) + c(4,4) \end{Bmatrix} + w(0,4)$$

(T_{04})

$$= \begin{Bmatrix} 0 + 19 \\ 8 + 8 \\ 19 + 4 \\ 25 + 0 \end{Bmatrix} + 16 = 32$$

$r(0,4) = k = 2$

T_{00}

$C(0,0) = 0$

$W(0,0) = 2$

$r(0,0) = 0$

T_{11}

$C(1,1) = 0$

$W(1,1) = 3$

$r(1,1) = 0$

T_{22}

$C(2,2) = 0$

$W(2,2) = 1$

$r(2,2) = 0$

T_{01}

$C(0,1) = 8$

$W(0,1) = 8$

$r(0,1) = 1$

T_{12}

$C(1,2) = 7$

$W(1,2) = 7$

$r(1,2) = 2$

T_{23}

$C(2,3) = 3$

$W(2,3) = 3$

$r(2,3) = 3$

T_{02}

$W(0,2) = 12$

$C(0,2) = 19$

$r(0,2) = 1$

T_{13}

$W(1,3) = 9$

$C(1,3) = 12$

$r(1,3) = 2$

T_{24}

$= 5$

$W(2,4) = 8$

$C(2,4) = 3$

$r(2,4)$

T_{03}

$W(0,3) = 14$

$C(0,3) = 25$

$r(0,3) = 2$

T_{14}

$W(1,4) = 11$

$C(1,4) = 19$

$r(1,4) = 2$

T_{04}

$W(0,4) = 16$

$C(0,4) = 32$

$r(0,4) = 2$

T_{33}

$c(3,3) = 0$

$w(3,3) = 1$

$r(3,3) = 0$

T_{44}

$c(4,4) = 0$

$w(4,4) = 1$

$r(4,4) = 0$

$r(0,4)$

T_{34}

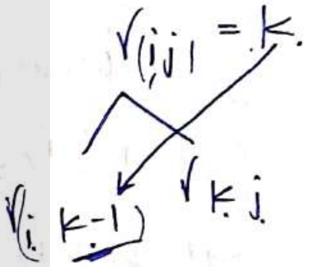
$c(3,4) = 3$

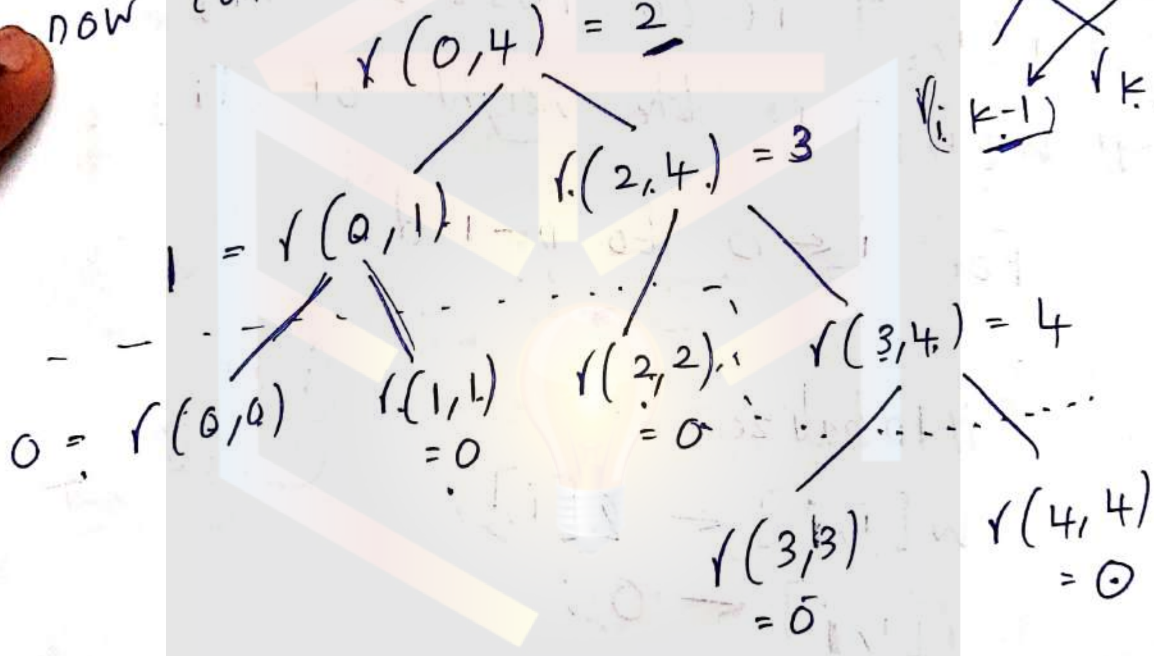
$w(3,4) = 3$

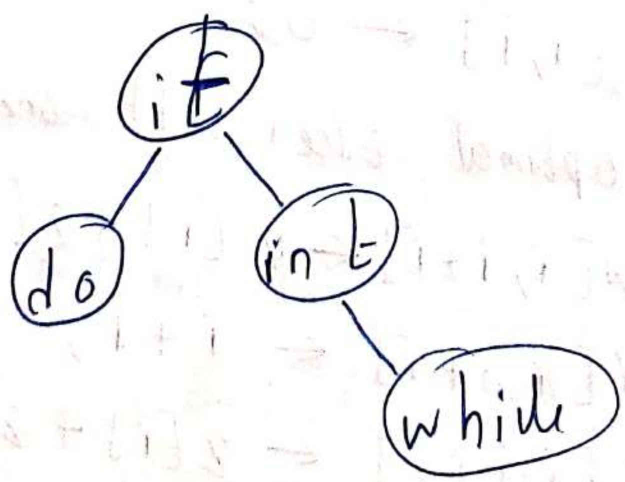
$r(3,4) = 4$

T_{04}

now construction of tree







→ Dec 2017

16) a) — same problem

→ April/May 2019, May/June 2018

14 ↗ 14 ↗

SN unit 4

→ April/May 2019

ii) 15, 8, 20, 12, 9, 1, 34, 4

⇓

→ 0/1 Knapsack using Backtracking:

→ Same problem which we discussed previously

→ Here we solve the same problem using concept of Backtracking.

ex: P = (11, 21, 31, 33, 43, 53, 55, 65)

W = (1, 11, 21, 23, 33, 43, 45, 55)

M = 110 solve using 0/1 knapsack

n = 8 problem using backtracking

= Give write same above data

make the data is in decreasing order of (P_i/W_i)

→ (P_i/W_i) = (11, 1.90, 1.47, 1.43, 1.30, 1.23, 1.22, 1.18)

(8 steps process as 8 items).

Step 1:

k = 1 $C_p = 0$ $C_w = 0$

↑ increasing as step 1 is the 1st value/item

↑ profit till now as per selected

↑ weight till now as per selected

Check $\hat{C}_w + w[k] \leq m$ ← max weight.

$0 + 1 \leq 100$ (True) [y[1] = 1]

Check $(k < n) \Rightarrow (1 < 8)$ ✓

continue to next step (2, 11, 1)

Step 2:

k = 2 C_p = 11 C_w = 1

Check $C_w + w[k] \leq m$

$(1 + 11 \leq 110)$ True [y[2] = 1]

check $(k < n) \Rightarrow (2 < 8)$ ✓

continue to next step (3, 32, 12)

Step 3:

k = 3 C_p = 32 C_w = 12

check $C_w + w[k] \leq m$

$(12 + 21 \leq 110)$ True [y[3] = 1]

check $(k < n) \Rightarrow (3 < 8)$ ✓

continue to next step (4, 63, 33)

0/1 GA — DR

GA 0/1

P_i/w_i

Step 4

$k = 4$ $\quad C_p = 63$ $\quad C_w = 33$

Check if $((C_w + w[k] \leq m)$

$33 + 23 \leq 110$ (True) $[y[4] = 1]$

check $(k < n) \Rightarrow 4 < 8$ ✓

next step $(5, 96, 56)$

Step 5

$k = 5$ $\quad C_p = 96$ $\quad C_w = 56$

check if $((C_w + w[k] \leq m)$

$56 + 33 \leq 110$ (True) $[y[5] = 1]$

check $(k < n) \Rightarrow 5 < 8$ ✓

$(5 < 8)$ next step $(6, 139, 89)$

Step 6

$k = 6$ $\quad C_p = 139$ $\quad C_w = 89$

89
43
≥ 110

Check if $(C_w + w[k] \leq m)$

$89 + 43 \leq 110$ (False)

$\therefore Y[6] = 0$

(89, 42, 132)

$(6 < 8)$ next step $(7, 139, 89)$

Step 7

$k = 7 \quad C_p = 139 \quad C_w = 89$

check $((C_w + w[k] \le m)$

$89 + 45 \le 110$ (false)

$\therefore Y[7] = 0$

$(7<8)$ next step $(8, 139, 89)$

Step 8

$k = 8 \quad C_p = 139 \quad C_w = 89$

Check $((C_w + w[k] \le m)$

$89 + 55 \le 110$

$144 \le 100$

check $(k < n)$ $(8 < 8)$ False

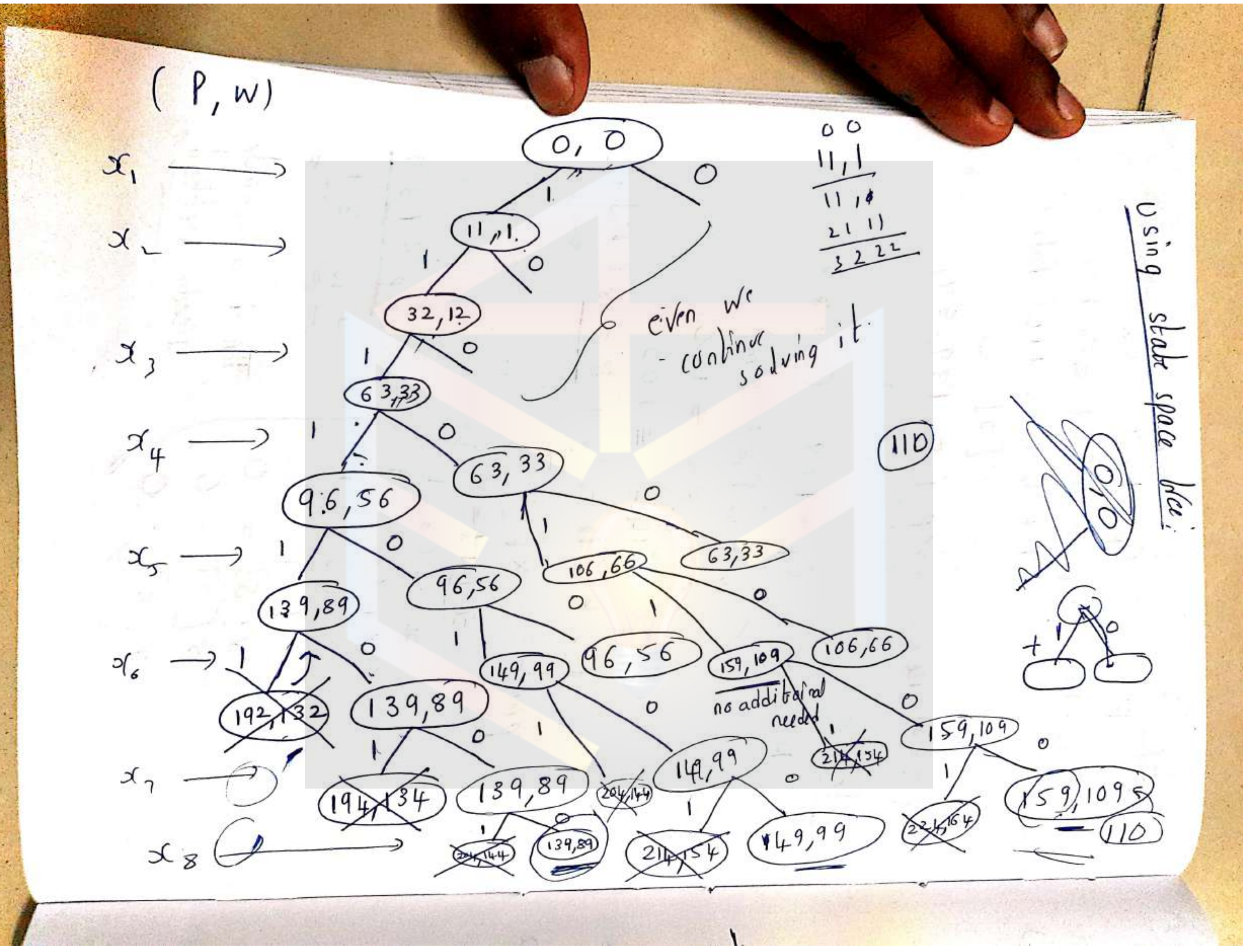
$\therefore$ print $x[1:n]$

$x[1] = 1$
$x[2] = 1$
$x[3] = 1$
$x[4] = 1$
$x[5] = 1$
$x[6] = 0$
$x[7] = 0$
$x[8] = 0$

$C_p = 139$

$C_w = 89$

This is a simple greedy method.

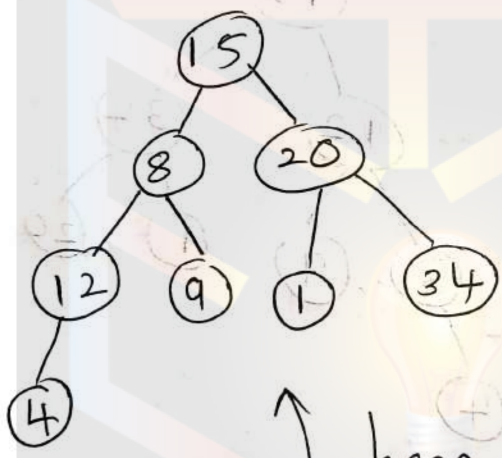

(P, W)
Using state space tree:
0, 0
11, 1
32, 12
63, 33
96, 56
139, 89
192, 132
194, 134
106, 66
149, 99
159, 109
110
even we continue solving it.
no additional needed

SN unit 4

April/May 2019

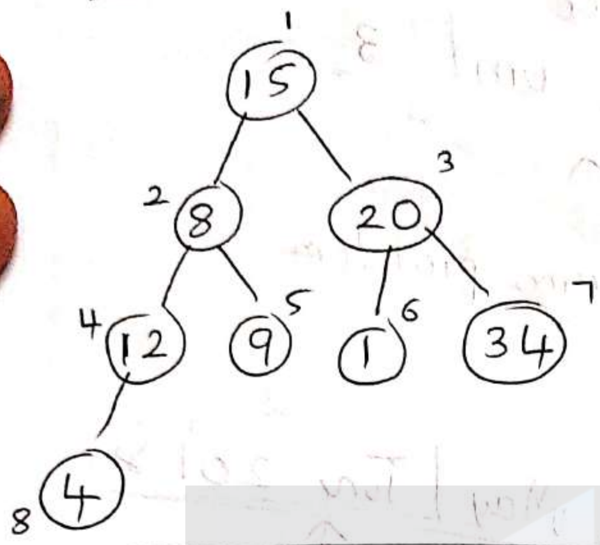
ii) 15, 8, 20, 12, 9, 1, 34, 4

⇓



↑ heap

building max heap :-



1 to 4

⇓

@ 4

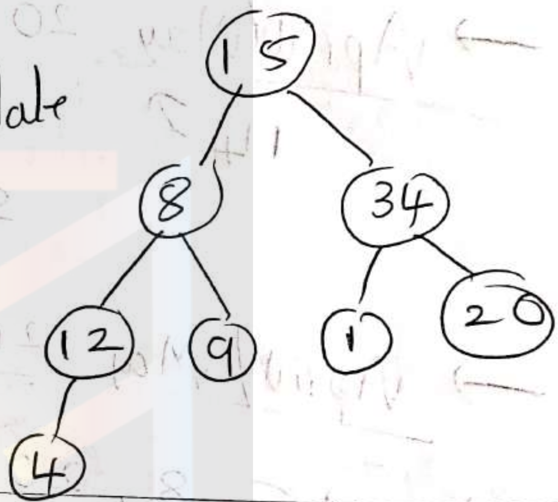
no swap.

as 12 is max at as root

@ 3

20 > 1 ✓ 20 > 34 ✗

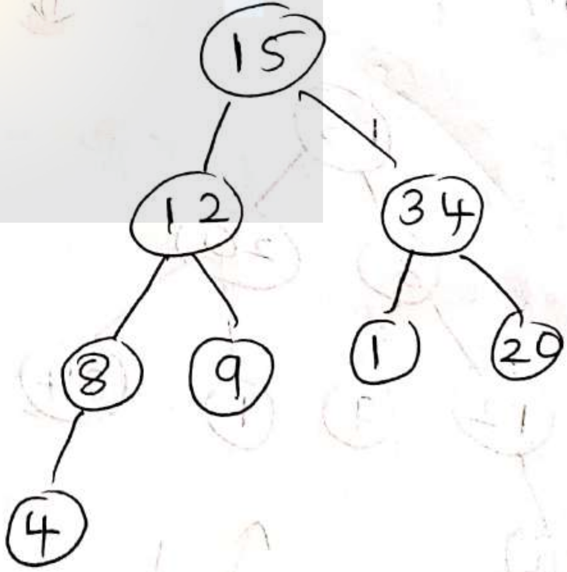
34 max ⟹ update



@ 2

8 > 12 12 > 9

12 max ⟹ update

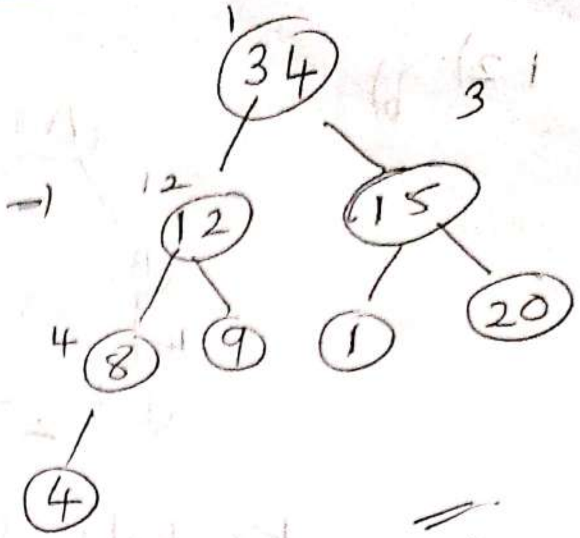


@ 1

15 > 12 ✓

15 > 34 ✗

max 34 →



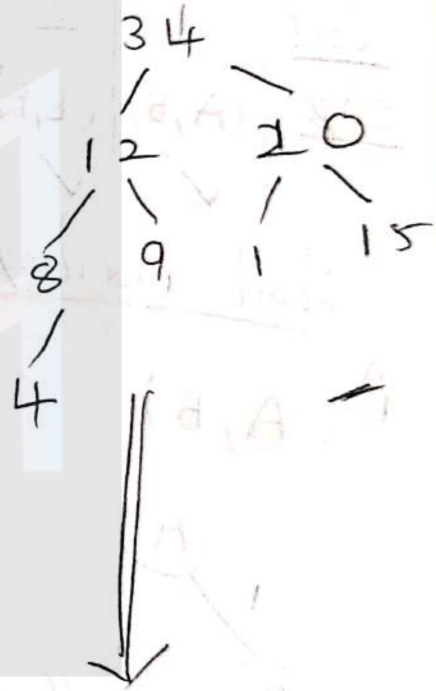
next iteration (write step wise as per marks)

@ 4 ⇒ no change

@ 3 ⇒ 20 is max ⇒

@ 2 ⇒ no change

@ 1 ⇒ no change

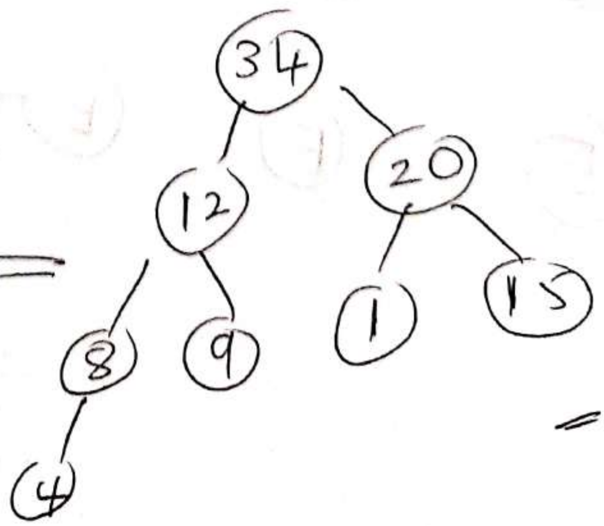


next iteration

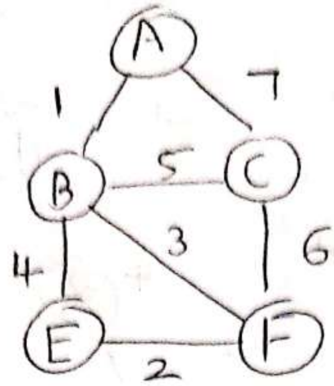
@ 4
@ 3
@ 2
@ 1

} no change

max heap ⟸



12) b)



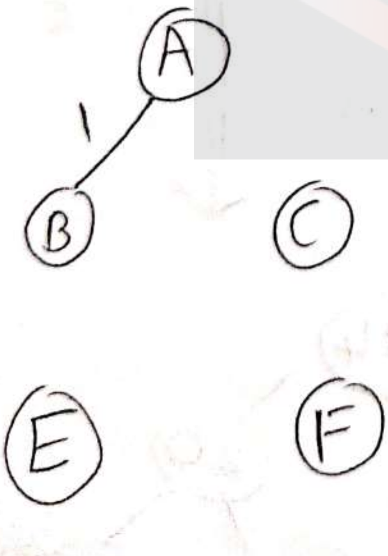
using Kruskal's algorithm

(sorted)

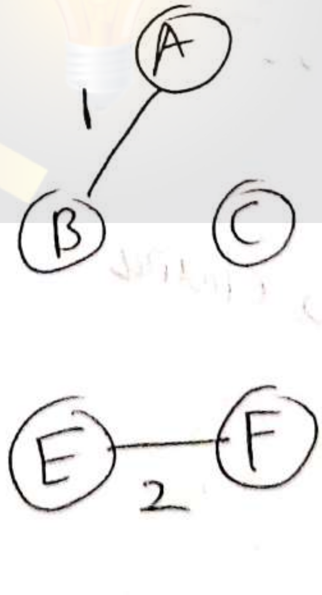
Cost	1	2	3	4	5	6	7
edge	(A,B)	(E,F)	(B,F)	(B,E)	(B,C)	(C,F)	(A,C)
	✓	✓	✓	✗			

start inserting

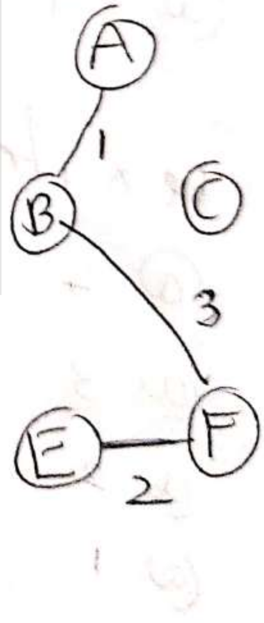
i) (A, B)



ii) B, F



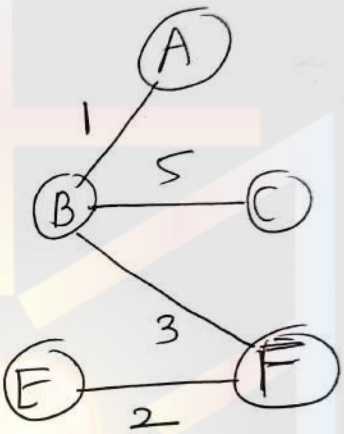
iii) B F



iv) BE

will cause loop so avoid

v) BC



all vertices visited so stop

∴ cost ⇒ 1 + 2 + 3 + 5

⇒ 11

→ May 2017

14)

∞	7	3	12	8
3	∞	6	14	9
5	8	∞	6	18
9	3	5	∞	11
18	14	9	8	∞

row reduct

∞	7	3	12	8	3
3	∞	6	14	9	3
5	8	∞	6	18	5
9	3	5	∞	11	3
18	14	9	8	∞	8
					22

⇒

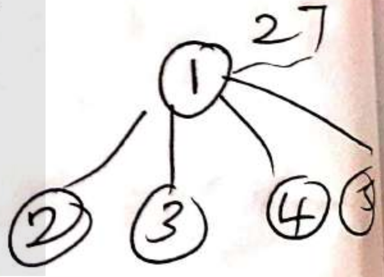
∞	4	0	9	5
0	∞	3	11	6
0	3	∞	1	13
6	0	2	∞	8
10	6	1	0	∞

max ← A[i]

$$\begin{bmatrix} \infty & 4 & 0 & 9 & 5 \\ 0 & \infty & 3 & 11 & 6 \\ 0 & 3 & \infty & 1 & 13 \\ 6 & 0 & 2 & \infty & 8 \\ 10 & 6 & 1 & 0 & \infty \end{bmatrix} \rightarrow \begin{bmatrix} \infty & 4 & 0 & 9 & 0 \\ 0 & \infty & 3 & 11 & 1 \\ 0 & 3 & \infty & 10 & 8 \\ 6 & 0 & 2 & \infty & 3 \\ 10 & 6 & 1 & 0 & \infty \end{bmatrix}$$

0 0 0 0 5 | 5

$22 + 5 = 27$



Possible path

row 1 = ∞

(3,1) = ∞)

Path (1, 2)   row = 1 (∞)   col = 2 (∞)

@ (2, 1) = ∞

row

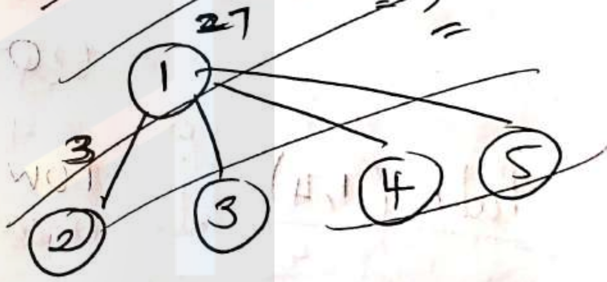
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 3 & 11 & 1 \\ 0 & \infty & \infty & 1 & 8 \\ 6 & \infty & 2 & \infty & 3 \\ 10 & \infty & 1 & 0 & \infty \end{bmatrix} \begin{matrix} 0 \\ 1 \\ 0 \\ 2 \\ 0 \\ \hline 3 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 10 & 0 \\ 0 & \infty & \infty & 1 & 8 \\ 4 & \infty & 0 & \infty & 1 \\ 10 & \infty & 1 & 0 & \infty \end{bmatrix}$$

col

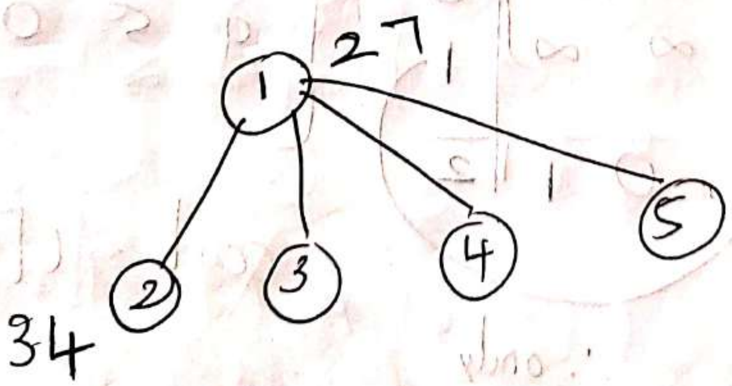
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 10 & 0 \\ 0 & \infty & \infty & 1 & 8 \\ 4 & \infty & 0 & \infty & 1 \\ 10 & \infty & 1 & 0 & \infty \\ \hline 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{no change}$$

$r = 3 + 0$
$= 3$

~~⇒ 0 + 3   3 + 0   = 3~~



∴ cost = C[1,2] + C[1] + r
= 4 + 27 + 3 = 34



Path (1, 3)

row 1 = ∞, col 3 = ∞, (3,1) = ∞

					row red.
∞	∞	∞	∞	∞	0
0	∞	∞	11	1	0
∞	3	∞	1	8	1
6	0	∞	∞	3	0
10	6	∞	0	∞	0
0	0	0	0	1	**2**

∞	∞	∞	∞	∞
0	∞	∞	11	0
∞	2	∞	0	6
6	0	∞	∞	2
10	6	∞	0	∞

r = 1

$cost = C[1,3] + C(1) + r$
$= 0 + 27 + 2 = 29$

Path (1, 4)

row 1 = ∞, col 4 = ∞, (4,1) = ∞

					row red.
∞	∞	∞	∞	∞	0
0	∞	3	∞	1	0
0	3	∞	∞	8	0
∞	0	2	∞	3	0
10	6	1	∞	∞	1
0	0	1	0	1	2

(the column 3 entry “1” →) no need as row 5 (1) will make it 0 ∴ only r = 2

∞	∞	∞	∞	∞
0	∞	3	∞	0
0	3	∞	∞	7
∞	0	2	∞	2
9	5	0	∞	∞

$cost = C[1,4] + C(1) + r$
$= 9 + 27 + 2$
$= 38$

Path (1, 5)

row 1 = ∞ row 5 = ∞

(5, 1) = ∞

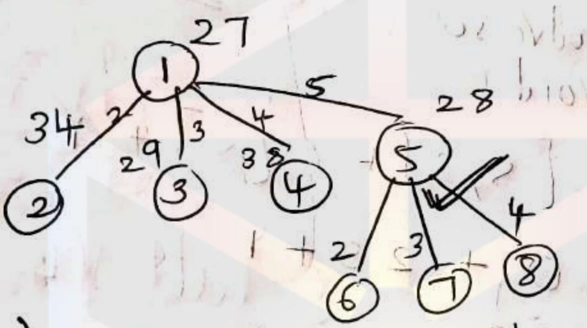
∞	∞	∞	∞	∞	0
0	∞	3	11	∞	0
0	3	∞	1	∞	0
6	0	2	∞	∞	0
∞	6	1	0	∞	0
0	0	1	0	∞	1

=>

∞	∞	∞	∞	∞
0	∞	2	11	∞
0	3	∞	1	∞
6	0	1	∞	∞
∞	6	0	0	∞

cost = c[1, 5] + c[1] + r

= 0 + 27 + 1 = 28



left path

(5, 2) (5, 4)

(5, 3)

Note use Path (1,5) matrix

Path (5, 2)

row 5 = ∞ col 2 = ∞ (2,1) = ∞

∞	∞	∞	∞	∞	0
0	∞	2	11	∞	0
0	∞	∞	11	∞	0
6	∞	1	∞	∞	1
∞	∞	∞	∞	∞	0
0	0	0	●	0	2

1 1

that will resolve so avoid this

=>

∞	∞	∞	∞	∞
0	∞	2	11	∞
0	∞	∞	0	∞
5	∞	0	∞	∞
∞	∞	∞	∞	∞

cost = c[5, 2] + c[5] + r

= 6 + 28 + 2

= 36

if (i -0) ...

max ←

Path (5,3)

row 5 = ∞, col 3 = ∞, (3,1) = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \\ \infty & 3 & \infty & 1 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 & 1 & 0 & \vert\, 1 \end{matrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \\ \infty & 2 & \infty & 0 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

that will resolve so avoid

$r = 1$

$cost = C[5,3] + C[5] + r$

$= 0 + 28 + 1$

$= 29$

Path (5,4)

row 5 = ∞, row4 = ∞, (4,1) = ∞

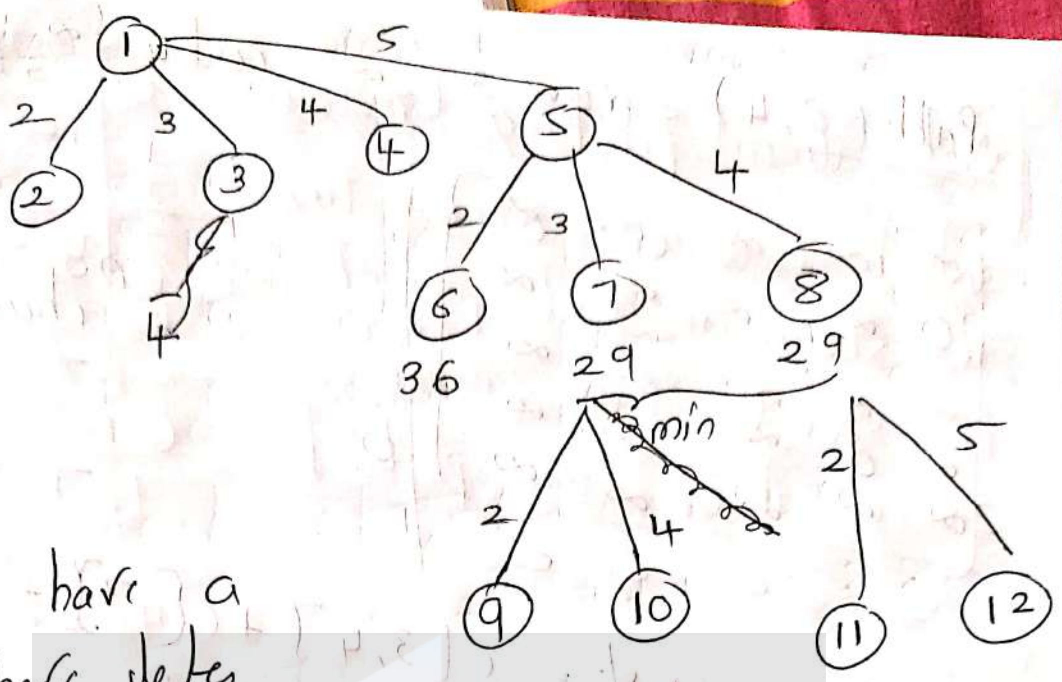
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ 6 & 0 & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 1 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ 6 & 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 1 & 0 & 0 & \vert\, 1 \end{matrix}$$

$cost = C[5,4] + C[5] + r$

$= 0 + 28 + 1$

$= 29$



as we have a clash here, let's solve both. (Use Path 53)

row 3, col 2, (2,1) = ∞

Path (3, 2)

∞	∞	∞	∞	∞	0
∞	∞	∞	11	∞	11
∞	∞	∞	∞	∞	0
∞	∞	∞	∞	∞	0
6	0	∞	∞	∞	0
∞	∞	∞	∞	0	
6	0	0	11	0	17

will be solved so avoid

∞	∞	∞	∞	∞
∞	∞	∞	0	∞
∞	∞	∞	∞	∞
∞	∞	∞	∞	∞
0	0	∞	∞	∞
∞	∞	∞	∞	

r = 17

cost = c[3, 2] + c[7] + r

= 2 + 29 + 17

= 48

31
17
48

Path (3, 4)   row 3 = ∞   col 4 = ∞   (4, 1) = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

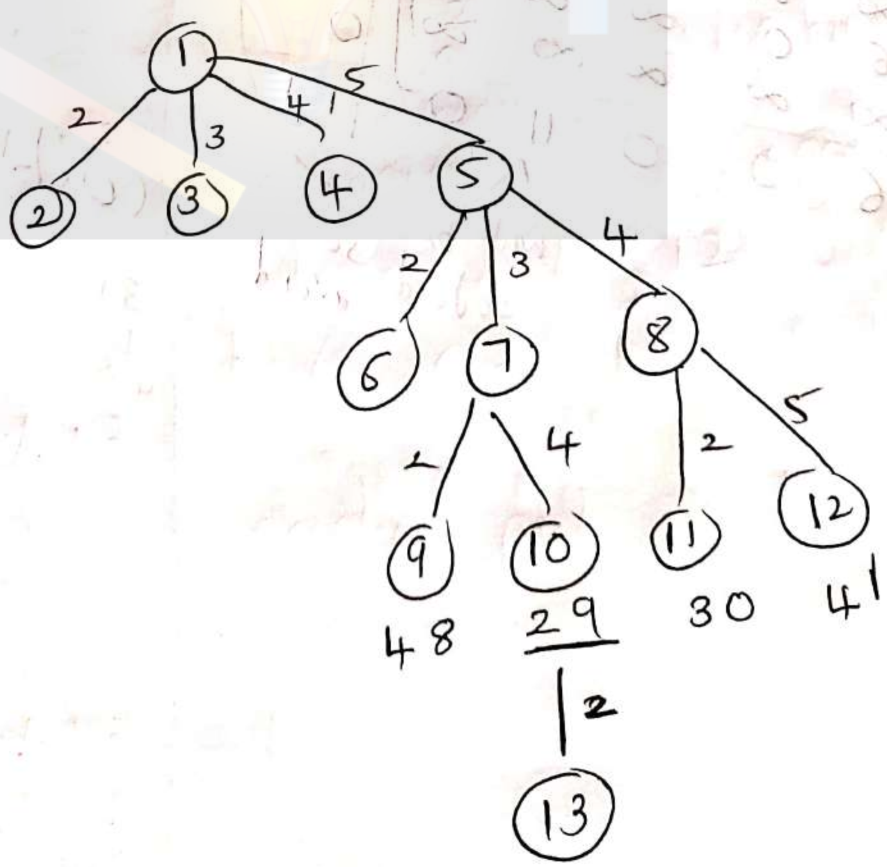
$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix}$$

⇒ no change

$$\text{cost} = C[3,4] + C[7] + r$$
$$= 0 + 29 + 0 = \underline{29}$$

continue Path (4, 2) & Path (4, 5)

u will get → 30   → 41



Path (4, 2) row 4 = ∞ ~~row~~ col 2 = ∞

(2, 1) = ∞

∞	∞	∞	∞	∞	0
0	∞	∞	∞	∞	0
∞	∞	∞	∞	∞	0
∞	∞	∞	∞	∞	0
∞	∞	∞	∞	∞	0
0	0	0	0	0	

$\therefore$ cost --- $C[4, 2] + C[10] + r$

$= 0 + 29 + 0$

$= 29.$

$\therefore$ Path

1 → 5 → 3 → 4 → 2 = 29 ← cost

→ Nov/Dec 2016

11) b) Linear probing bucket = 13 → b = 13

$F(K) = K \% b$

Key ⇒ 7, 42, 25, 70, 14, 38, 8, 21, 34, 11

$\therefore F(K) = K \% 13$

[Note: linear probing means which collision occurs then its stored in next closest free spot (collision means 2 keys at a same spot)]

Index	Key
0	38
1	14
2	
3	42
4	
5	70
6	
7	7
8	8
9	21
10	34
11	11
12	25
13	//////

↑ will be update step by step

For 7

$F(k) = 7 \% 13$

$= 7$

For 42

$F(k) = 42 \% 13$

$= 3$

For 25

$F(k) = 25 \% 13$

$= 12$

For 70

$F(k) = 70 \% 13$

$= 5$

For 14

$= 14 \% 13$

$= 1$

For 38

$F(k) = 38 \% 13$

$= 12$ ← already occupied

hence after spot 12 we have 0 spot as 0 is empty put 38 in 0 f(...) = min () then

For 8

F(K) = 8 % 13
= 8

For 21

F(K) = 21 % 13
= 8
↑ already occupied hence next spot
9 is free so put into 9

For 34

F(K) = 21 % 13
= 8
↑ already occupied hence next spot
9 is also occupied next spot 10
↑
so push int 10

insertion done.

For 11

F(K) = 11 % 13
= 11

→ as it has high (twice) collision

max no ⇒ For 34 we need to check 8, 9, 10
so 3 checks max

avg ⇒ in order write all no. of matches

for	7	42	25	70	14	38	8	21	34	11
matches	1	1	1	1	1	2	1	2	3	1

So $= \frac{1+1+1+1+1+2+1+2+3+1}{10}$

$= \frac{14}{10} = 1.4$ → avg

→ among all min

min = 1

max = 3

↑ as per previous logic

Dec 2017

13) b)

in short note unit 3

exi: 3 device D_1, D_2 & D_3
& cost 30, 15, 20
max cost = 105
realiability = 0.9, 0.8, 0.5 respectively.

→ given

$C = 105$

$C_1 = 30,\ C_2 = 15,\ C_3 = 20$

$r_1 = 0.9,\ r_2 = 0.8,\ r_3 = 0.5$

105
−15
−20
70

70 / 30 = 2.33 = ⌊2⌋ = 2

$$U_i = \left\lfloor \frac{C + C_i - \sum_{j=1}^{n} C_j}{C_i} \right\rfloor$$

$$U_1 = \left\lfloor \frac{C + C_1 - (C_1 + C_2 + C_3)}{C_1} \right\rfloor$$

$$= \left\lfloor \frac{105 + 30 - (30 + 15 + 30)}{30} \right\rfloor$$

$$= \left\lfloor \frac{135 - 65}{30} \right\rfloor = \left\lfloor \frac{70}{30} \right\rfloor = \lfloor 2.33 \rfloor = 2$$

$$U_2 = \left\lfloor \frac{C + C_2 - (C_1 + C_2 + C_3)}{C_2} \right\rfloor = \left\lfloor \frac{105 + 15 - 65}{15} \right\rfloor$$

$$= \left\lfloor \frac{120 - 65}{15} \right\rfloor = \lfloor 3.66 \rfloor = 3$$

$$U_3 = \left\lfloor \frac{C + C_3 - (C_1+C_2+C_3)}{C_3} \right\rfloor = \left\lfloor \frac{105+20-65}{20} \right\rfloor$$

$= \lfloor 3 \rfloor$

$= 3$.

$\therefore U_1 = 2$

$U_2 = 3 \Rightarrow$ ✓

$U_3 = 3$

same set's concept like 0/1 knapsack

$S^0 = (r_1, C)$ — r: realiability, C: cost

initially realibility = 100% = 1 as we have no device hence cost = 0

$\therefore S^0 = \{(1, 0)\}$

Consider Device 1 (D_1)

$1 \le m_i \le U_i \Rightarrow 1 \le m_1 \le 2$

$m = 1$ or $m = 2$

m = 1

(S_j^i) $S_1^1 \Rightarrow i = 1, j = 1, m = 1$

$\phi_1(m_1) = 1 - (1 - r_1)^{m_1}$

$= 1 - (1 - 0.9)^1$

$= 1 - 0.1$

$= 0.9$

$S_1^1 = \{ (0.9 * 1), (0 + 30) \}$ ← cost

$= \{ 0.9, 30 \}$

$\underline{m = 2}$

$S_2^1 \Rightarrow i = 1, j = 2, m_1 = 2$

$\phi(m_1) = 1 - (1 - 0.9)^{2}$

$= 1 - 0.01$

$= 0.99$

$S_2^1 = \{ 0.99 * 1, 0 + 60 \}$ ← (30 * 2)

$= \{ 0.99, 60 \}$

$S^1 = S_1^1 \cup S_2^1$

$= \{ (0.9, 30), (0.99, 60) \}$

Consider Device 2 (D_2)

$1 \leq m_i \leq U_i \Rightarrow 1 \leq m_2 \leq 3$

$m = 1, m = 2, m = 3$

$\underline{m = 1}$

$S_1^2 \Rightarrow i = 2, j = 1, m = 1$

$\phi_1(m_2) = 1 - (1 - r_2)^{m_2}$

$= 1 - (0.2)^1$

$= 0.8$

$S_1^2 = \{ (0.9 * 0.8, 30 + 15), (0.99 * 0.8, 60 + 15) \}$

$\{ \text{CALIS} \dots$

$= \{(0.72, 45), (0.792, 75)\}$

<u>m = 2</u>

$S_2^2 \rightarrow i = 2, j = 2, m_2 = 2$

$\phi_2(m_2) = 1 - (1 - r_2)^{m_2}$

$= 1 - (1 - 0.8)^2$

$= 0.96$

$S_2^2 = \{(0.9 * 0.96, 30 + 2 * 15), (0.99 * 0.96, 60 + 2 * 15)\}$

$= \{(0.864, 60), (0.9504, 90)\}$

<u>m = 3</u>

$S_3^2 \rightarrow i = 2, j = 3, m_2 = 3$

$\phi_2(m_2) = 1 - (1 - r_2)^{m_2}$

$= 1 - (1 - 0.8)^3$

$= 0.992$

$S_3^2 = \{(0.9 * 0.992, 30 + 3 \times 15), (0.99 \times 0.992, 60 + 3 \times 15)\}$

S^1_2

$= \{(0.8928, 75), (0.9208, 105)\}$

$S^2 = S_1^2 \cup S_2^2 \cup S_3^2$

$= \{(0.72, 45), (0.792, 75), (0.864, 60), (0.9504, 90), (0.8928, 75), (0.9208, 105)\}$

Now applying purging / Dominance rules

$(0.792, 75)$ $(0.864, 60)$

no need (remove)

$(0.9504, 90)$ is no in proper order of cost so remove.

$S^2 = \{(0.72, 45), \sout{(0.792, 75)}, (0.864, 60), (0.8928, 75), (0.9208, 105)\}$

Consider device 3 (D_3)

$1 \le m_i \le U_i \Rightarrow 1 \le m_3 \le 3$

$m_3 = 1, 2, 3.$

m = 1

$S_1^3 \Rightarrow i = 3, j = 1, m = 1$

$\phi_2(m_2) = 1 - (1 - r_3)^{m_3}$

$= 1 - (1 - 0.5)^1$

$= 0.5$

$S_1^3 \Rightarrow \{(0.5 \times 0.7, 45+20), (0.5 \times 0.864, 60+20), (0.8928 \times 0.5, 75+20), (0.9208 * 0.5, 105+20))$

↳ skip as cost exceeded.

$S_1^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$

m = 2

$S_2^3 \rightarrow i = 3, j = 2, m_3 = 2$

$\phi(m_3) = 1 - (1 - r_3)^{m_3}$

$= 1 - (1 - 0.5)^2$

$= 1 - (0.5)^2 = 0.75$

$S_2^3 = \{(0.75 * 0.72, 45 + (2 * 20)),$

$(0.75 * 0.864, 60 + (2 * 20)),$

$(0.8928 * 0.75, 75 + (2 * 20)),$ ← 75 + 40 = 115 (remove)

$(0.75 * 0.98208, 105 + (2 * 20))\}$ ↑ more remo[ve]

$S_2^3 = \{(0.54, 85), (0.648, 100)\}$

m = 3

$S_3^3 \Rightarrow i = 3, j = 3, m_3 = 3$

$\phi(m_3) = 1 - (1 - 0.5)^3$

$= 1 - 0.125$

$= 0.875$

$S_3^3 = \{(0.875 * 0.72, 45 + (3*20))$
$(0.875 * 0.864, 60 + (3*20))$ ×
$(0.875 * 0.8928, 75 + (3*20))$ ×
$(0.875 * 0.98208, 105 + (3*20))\}$ ×

$= \{(0.63, 105)\}$

$\therefore S^3 = S_1^3 \cup S_2^3 \cup S_3^3$

$= \{(0.36, 65), (0.432, 80),$
$(0.4464, 95), (0.54, 85),$
$(0.648, 100), (0.63, 105)\}$

remove ↗ (0.4464, 95)

95 order mis-

~~= {(0.36,~~

$= \{(0.36, 65), (0.432, 80),$
$(0.54, 85), (0.648, 100)$
$(0.63, 105)\}$

(0.648, 100)   (0.63, 105)
↑ less reliable but more cost hence remove.

$\therefore S^3 = \{(0.36, 65), (0.432, 80),$
$(0.54, 85), (0.648, 100)\}$
↑
Best.

{ if (A[i] > A[j])

$\Rightarrow (0.648, 100) \in S_2^3$

we got ↑ $\therefore$ $m_3 = 2$

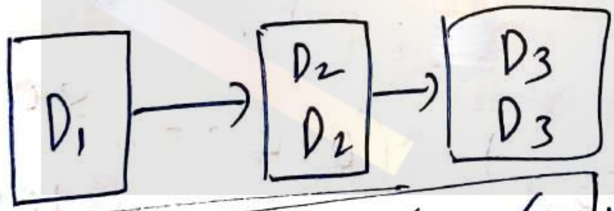
From $(0.864, 60) \in S_2^2$

we got ↑ $\therefore$ $m_2 = 2$

From $(0.9, 30) \in S_1^1$

$\therefore$ $m_1 = 1$

$\therefore (m_1, m_2, m_3) = (1, 2, 2)$



0.648 more reliable at cost 100 //

→ **Travelling Salesperson Problem :-** (14)

↳ Travelling salesmen problem consist of a salesman (Person) & a set of cities.