Unit-5: Planar graph & Digraph

→ A graph is planar graph if it is possible to represent it in the plane such that no edges or the graph intersect

→ A graph which cannot be drawn in a plane without crossover b/n its edges is called non planar graph

→ embedding is a drawing of geometric representation of a graph on any surface such that no 2 edges intersect

Embedding of planar graph G on a plane is called a planar representation of G

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→ A graph G drawn with crossing edge doesn't mean that G is non planar

→ An isomorphic graph of G that can be embedded in a plane is planar graph.



Representation of a planar graph :-

Planar graph embeded in a plane & it is called representation in the plane is called plane graph

Fary stein wagner theorme prove that any plain graph can be represented by straight



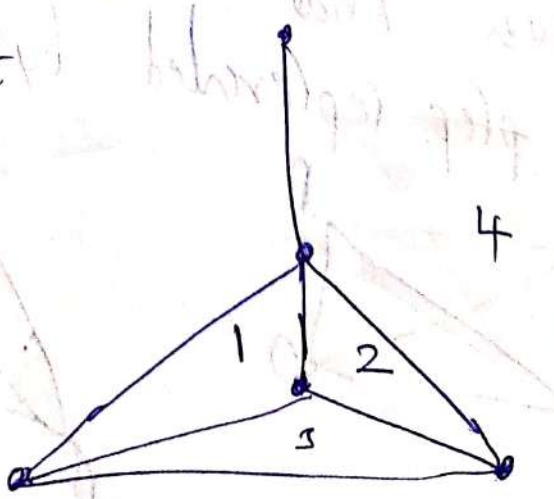
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region or face:

A plane representation of a graph divides the plain into region each region is characterized by the set of edges forming its boundary each plain graph determine a region of infinite area is called infinite region or exterior region.

Any other region is an interior region.

Boundary of region is the circuit that doesn't contain any cut edges is known as maximal circuit



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Degree of region:

No. of edges in a closed walk that encloses the region.
sum of degree of all regions
= 2 * no. of edges (size of the graph.

region 1, 2, 3 degree = 3
region 4 degree = 5 (3+2)
e counted twice.

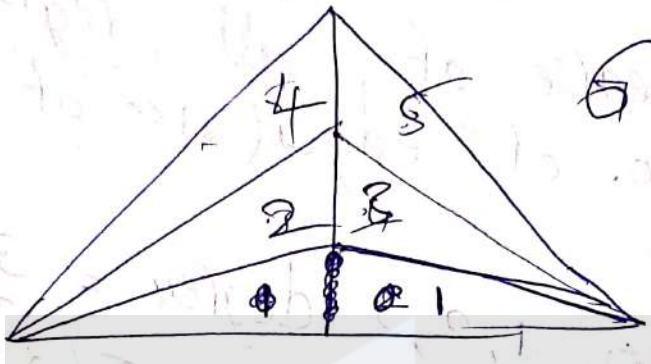
3 * 3 + 5 = 14
= 2 * 7 (no. of edges)

Maximal planar graph:

A simple planar graph is called a maximal planar graph if the graph becomes non planar when 2 non adjacent vertices in it are joined by an edge
→ it must be a connected graph.
→ Any planar graph is a spanning subgraph of maximum planar graph.

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A graph with degree of each region = 3 is called triangular

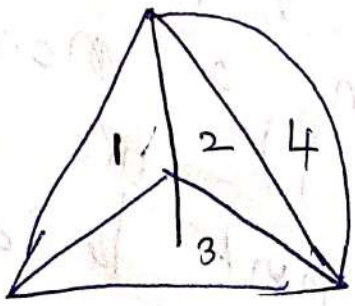


Euler's formula for planar graph:

A planar graph may have different planar representation no. of regions of all representation are same.

Theorem 1: In a connected planar graph G with n vertices & e edges has f regions then

$$n - e + f = 2$$



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i) if $e = 0$, $n = 1$, $f = 1$

then $n - e + f = 2$ (isolated vertex)

$1 - 0 + 1 = 2$

$2 = 2$ ✓

ii) Tree

$e \geq 1$, if G is a tree $n = e + 1$

& $f = 1$

$n - e + f = 2$

$e + 1 - e + 1 = 2$

$2 = 2$ ✓

iii) G is a graph & it has circuit

$G' = G - e'$ it has $e - 1$ edges

$n - (e-1) + f' = 2$

$n - e + 1 + f' = 2$

$n - e + f' = 1$

$x - e + (f - 1) = 1$

$x - e + f = 2$ ✓

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