Introduction :

A Graph (G) = (V, E) consist of V - A non empty set of vertices & E - A set of edges.

each edge has one or two vertices assosiated with it called its end points

An edge is said to connect its end points

$\frac{16000\cancel{0}}{6\cancel{0}0}$

Graph

Finite graph

Infinite graph

600 → 200

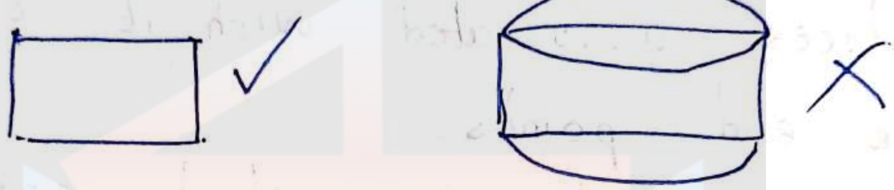
800 → ?

→ A graph with finite set of vertices is called finite graph &

→ A graph with infinite set of vertices is called infinite graph

$\frac{1600}{600}$

Simple Graph: A graph in which each edge connects two different vertices & where no two edges connect the same pair of vertices called simple graph



multi Graph: A graph that have multiple edges connecting same vertices are called multi Graph.

self loop: A edge that connects vertix to itself is called self loop

Pseudo Graph: Graph may include loops, multiple edges.

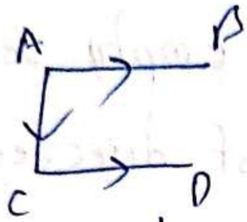
Directed Graph: A directed Graph (Di Graph) consist of non empty set of vertices V & a set of directed edges E

each directed edge is associated with an ordered pair of vertices (u, v) indicates (starts at u & ends at v).

A directed Graph with no loops & no multiple edges is called simple directed graph

A directed Graph with multiple directed edges is called directed multigraph.

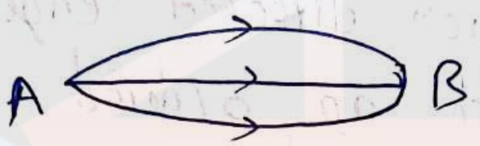
A Graph with booth directed & undirected edges is called mixed graph



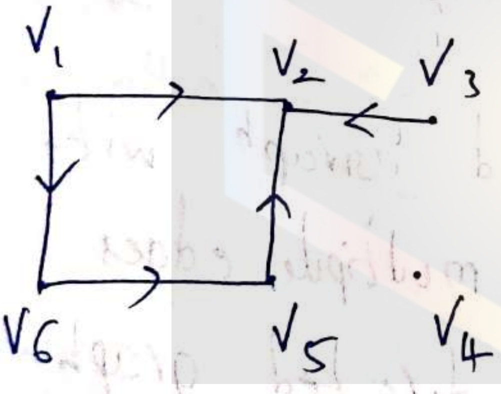
simple

directed loop

||ed directed edges



multiple directed graph.



× incoming ✓ ougoing	✓ incomi × out	× in × out
Source	Sink	isolated
V_1 V_3	V_2	V_4

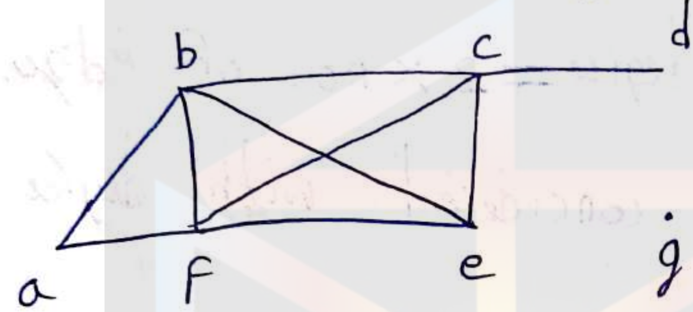
A vertex which is neither initial nor terminal is called as isolated vertex.

Degree of a vertex :

degree 0 - isolated
degree 1 - pented

The degree of a vertex in an undirected graph is the number of edges incident with it except that a loop

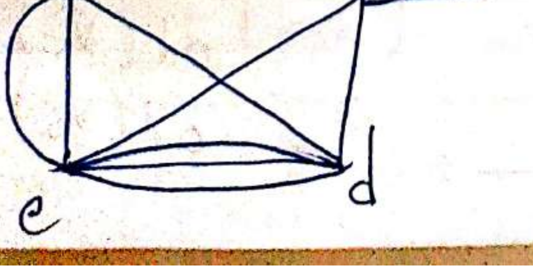
denoted by deg ( )



deg (a) = 2
deg (b) = 4
deg (c) = 4
deg (d) = 1
deg (e) = 3
deg (f) = 4
deg (g) = 0

A vertex with degree 1 is pendent vertex with degree 0 is isolated vertex

loop = 2 (undirected graph)



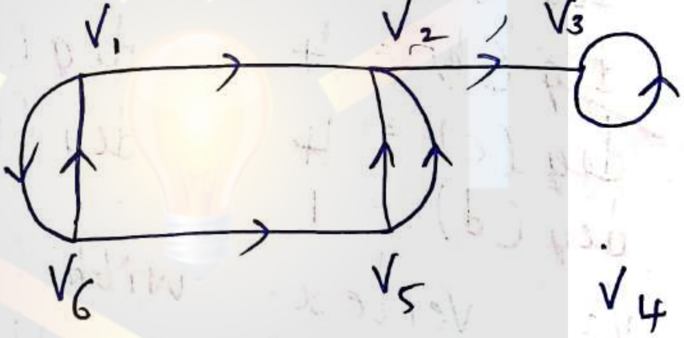
deg (a) = 4
deg (b) = 6
deg (C) = 1
deg (d) = 5
deg (e) = 6

$\underline{22} \Rightarrow 2 \times 11$

$\Downarrow$ no. of edges

degree = 2 × no. of edge.

loop is concidered with degree two



$d^-(V_1)$ / $id(V_1) = 1$ $\qquad$ $d^+(V_1)$ / $od(V_1) = 2$

$id(V_2) = 3$ $\qquad$ $od(V_2) = 1$

$id(V_3) = 2$ $\qquad$ $od(V_3) = 1$

$id(V_4) = 0$ $\qquad$ $od(V_4) = 0$

$id(V_5) = 1$ $\qquad$ $od(V_5) = 2$

$id(V_6) = 1$ $\qquad$ $od(V_6) = 2$

$\overline{8}$ $\qquad$ $\overline{8}$

$$id + od = 8 + 8$$
$$= 16$$
$$\Downarrow$$
$$2 \times \underbrace{8}_{\text{no of edges}}$$

Hand Shaking property!

Let $G = (V, E)$ be an undirected graph with e edges then

$$2e = \sum_{v} \deg(V)$$

→ Null graph is a graph or digraph with no edges

→ Order (no. of vertices).

→ Size (no. of edges).

→ Labelled graph (if we assign names to the vertices then the graph is called labelled graph)

→ Unlabelled graph (not assign names to vertical)

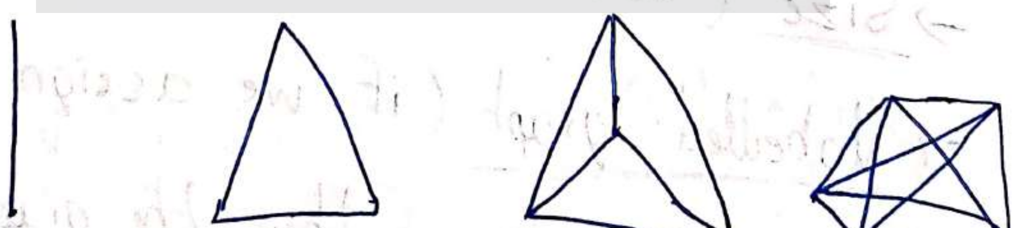
→ Two vertices are said to be adjacent if their is an edge joining them

→ <u>Complete graph</u> :

A simple graph of order greater than 2. in which their is an edge b/n every pair of vertices is called complete graph.

Denoted by K_n

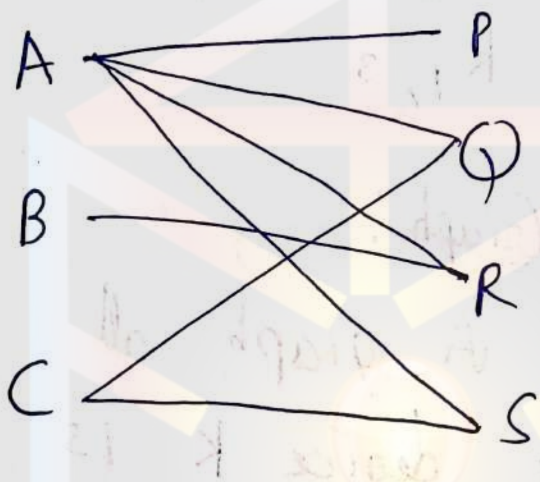
Complete graph is a simple graph in which every pair of distint vertices are adjacent



K_2 K_3 K_4 K_5

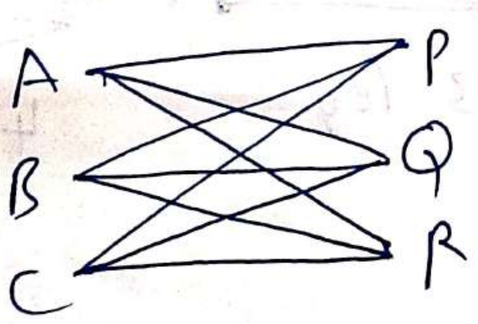
→ Bipartite graph:

A simple graph G is such that it's vertex set V is the union of two mutually disjoin non empty sets V_1 & V_2 which are such that every edge in G joins a vertex in V_1 & vertex in V_2

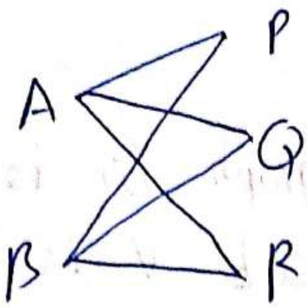


→ Complete Biparitite graph!

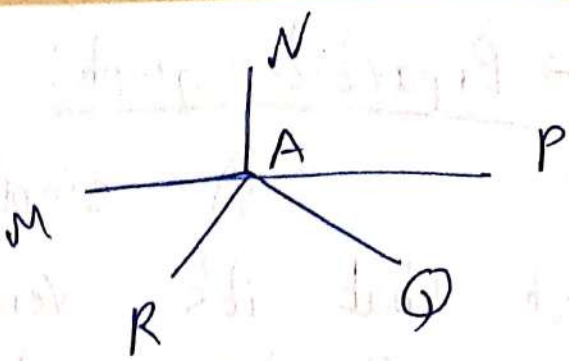
IF their is an edge b/n every vertex in V_1 & every vertex in V_2



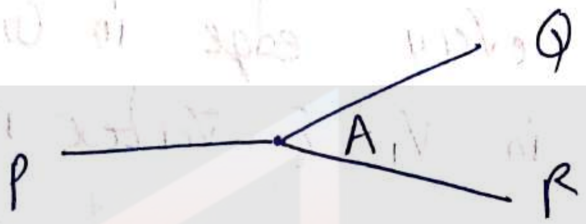
$K_{3,3}$



$K_{2,3}$



$K_{1,5}$

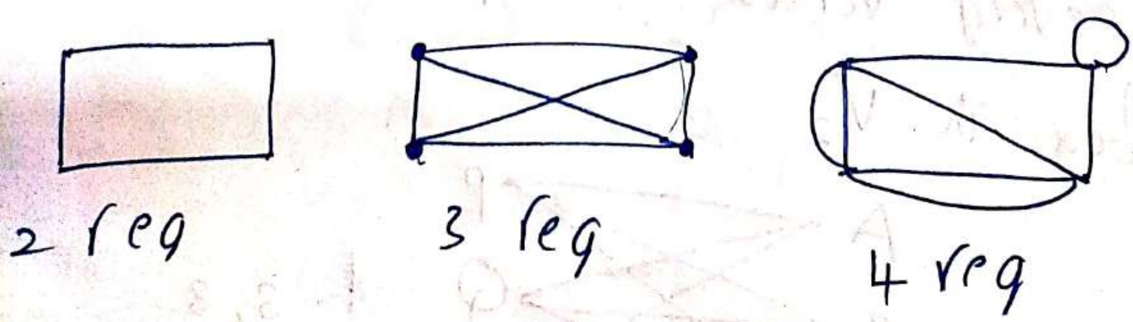


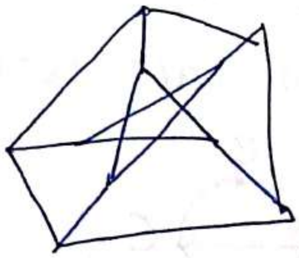
$K_{1,3}$

→ Regular Graph:

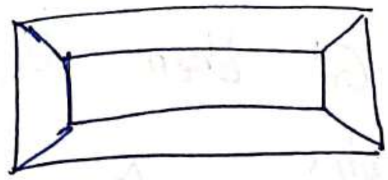
A graph all vertices are of same degree K is called regular graph of degree K

K reg graph





3 reg



3 dimensional hyper cube /cubic graph.

Representing Graphs & Graph Isomorphism

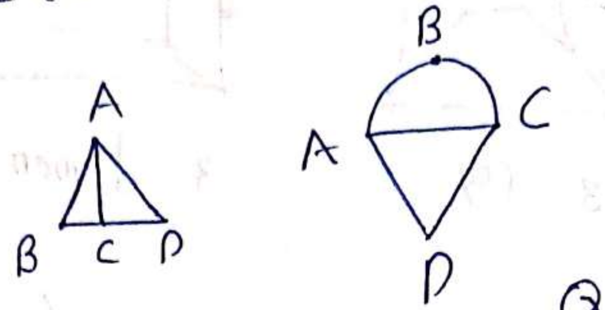
Consider 2 graphs $G = (V, E)$ & $G' = (V', E')$ suppose there exist a function $f : V \to V'$ such that

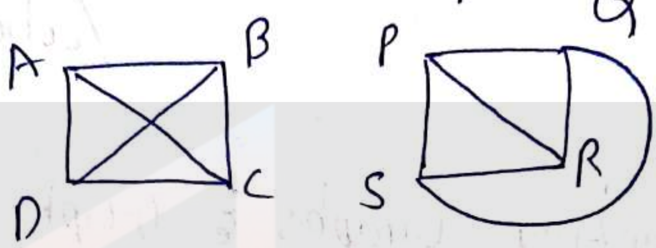
→ f is one to one and onto

→ for all vertices of $A, B (G)$ the edge $\{A, B\} \in E$ if and only if the edge $\{f(A), f(B)\} \in E'$

F is called isomorphism b/n G & G' then G & G' are isomorphic graphs

ex:-





<u>Note</u>:- Same no. of vertices, same no. of edges, an equal no. of vertices with given degree

Non Isomorphic

<u>Isomorphism of Digraph</u>:-

One to one correspondence b/n their vertices & edges such that adjacency of vertex along with directions in preserved.

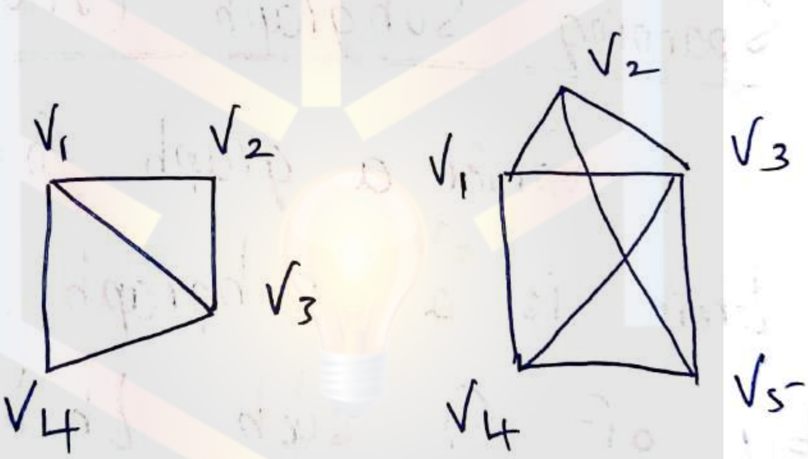
Sub Graph:

Two graphs G & G' we say that G_1 is sub graph of G if the following conditions hold

→ All the vertices and all edges of G_1 are in G

→ each edge of G_1 has the same end vertices in G as in G_1

ex)



→ Every graph is a sub graph of itself.

→ Every simple graph with n vertices is subgraph of complete graph K_n

→ G_1 is a subgraph of G_2 & G_2 is a subgraph of G. G_1 is subgraph of G.

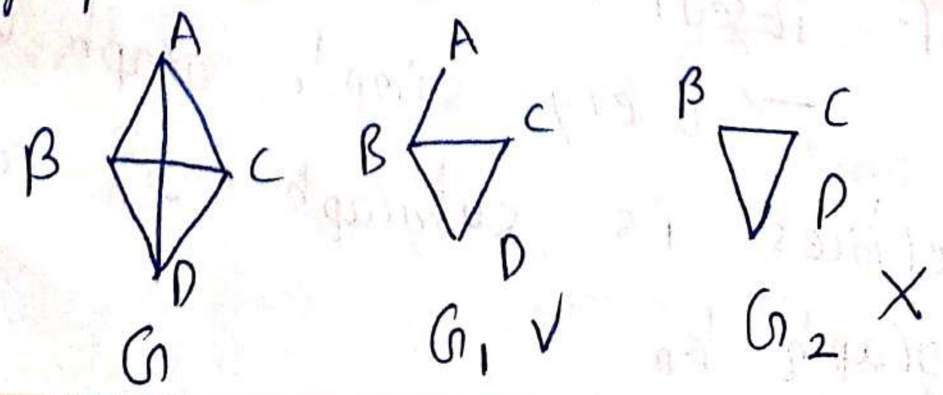
→ Single Vertex in G is a subgraph

→ Single Edge in G together with it's end vertices is sub graph of G.

Spanning Subgraph (All vertices)

Given a graph $G = (V, E)$ if there is a subgraph $G_1 = (V_1, E_1)$ of G such that $V_1 = V$ then G_1 is called Spanning sub graph of G

ex1

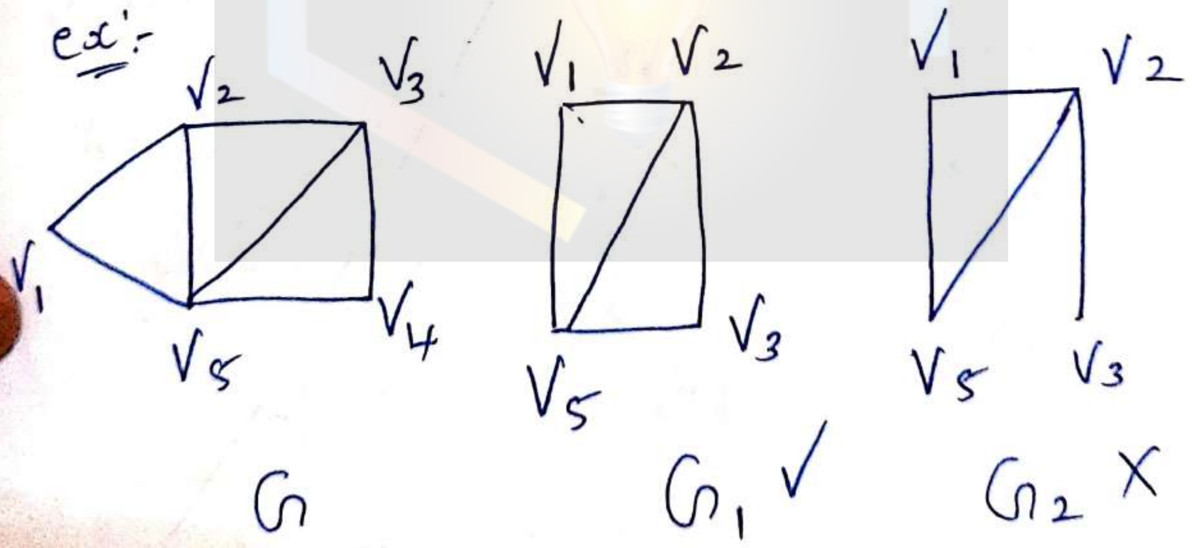


Induced SubGraph :- (All edges).

Given graph $G = (V, E)$

suppose there is subgraph $G_1 = (V_1, E_1)$ of G such that every edge $\{A, B\}$ where $A, B \in V_1$ is an edge of G_1 also then G_1 is called a Subgraph of G induced by V_1 denoted by $\langle V_1 \rangle$

ex:-



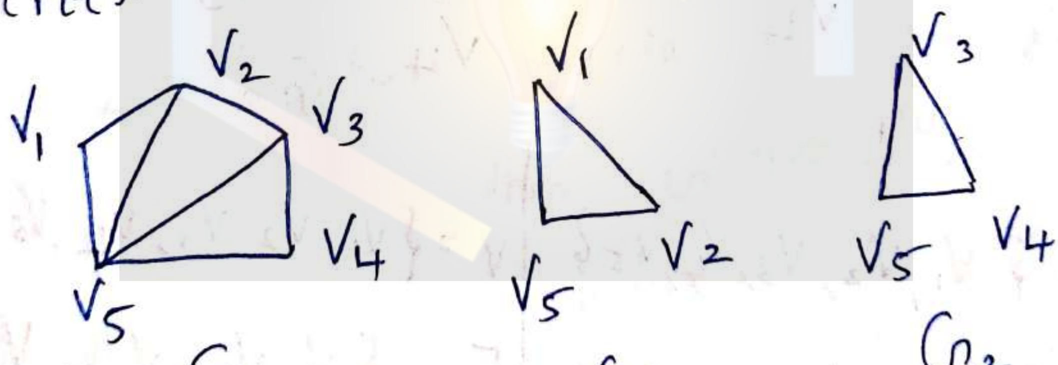
Edge disjoint & Vertex disjoint Sub Graph:

Let G be a graph & G_1 & G_2 be two subgraphs of G

then

→ G_1 & G_2 are edge disjoint if they ~~b~~ don't have any common edge.

→ G_1 & G_2 are vertex disjoint if they don't have any common vertex



G G_1 G_2

edge disjoint.

Operations of (on) Graphs :

$$G(V, E)$$

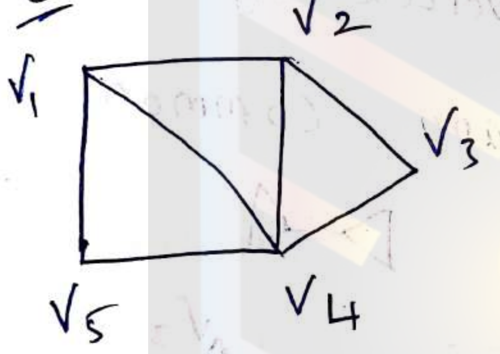
$$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$$

$$G_1 \cap G_2 = (V_1 \cap V_2, E_1 \cap E_2)$$

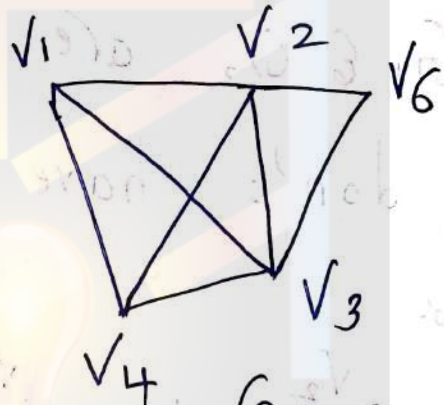
$$G_1 \Delta G_2 = (V_1 \cup V_2, E_1 \Delta E_2)$$

$$E_1 \Delta E_2 = (E_1 \cup E_2) - (E_1 \cap E_2)$$

ex:



G_1



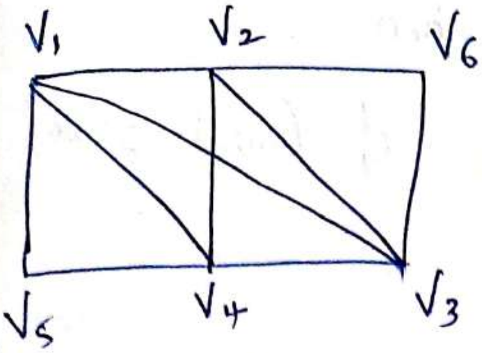
G_2

$V = \{V_1, V_2, V_3, V_4, V_5\}$

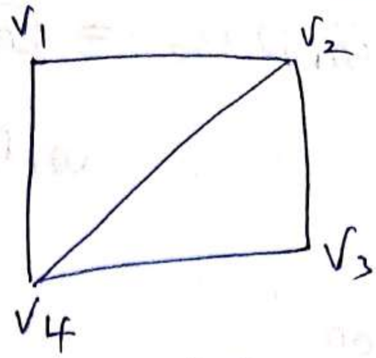
$E = \{V_1V_2, V_1V_4, V_1V_5, V_5V_4, V_2V_4, V_2V_3, V_4, V_3\}$

$V = \{V_1, V_2, V_3, V_4, V_6\}$

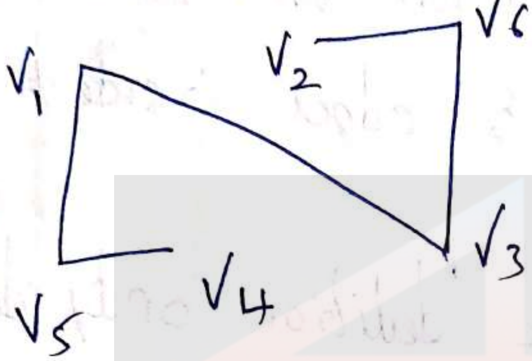
$E = \{V_1V_2, V_1V_3, V_1V_4, V_2V_4, V_2V_3, V_2V_6, V_6V_3, V_4V_3\}$



$G_1 \cup G_2$



$G_1 \cap G_2$



$G_1 \Delta G_2$

$G_1 = (V_1, E)$ the $G_1 \cup G_2$ is

$G_2 = (V_2, E_2)$ unions of V's & E's

then $G_1 \cap G_2$ is

intersection of V's & E's

then $G_1 \Delta G_2$ is union of vertices,

symetric difference of edges

then Symetric difference of edges is

union of E's − intersection of

edges's

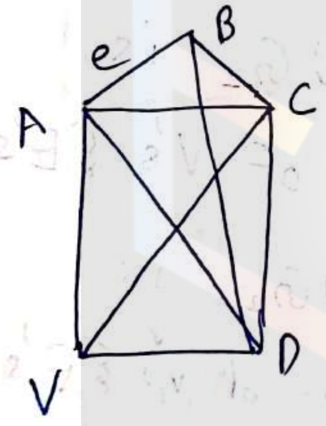
→ if $G_1 \cup G_2 = G$ then

$G_1 \cap G_2 = \phi$ (null graph)

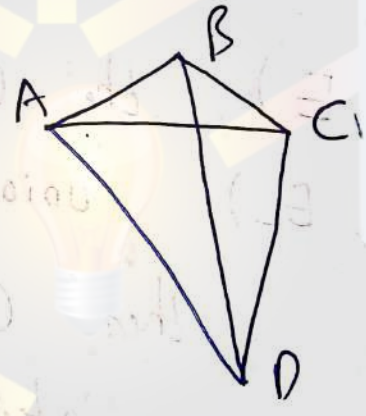
Delition :

In vertex delition we have to delete the vertex & edges incident on that vertex.

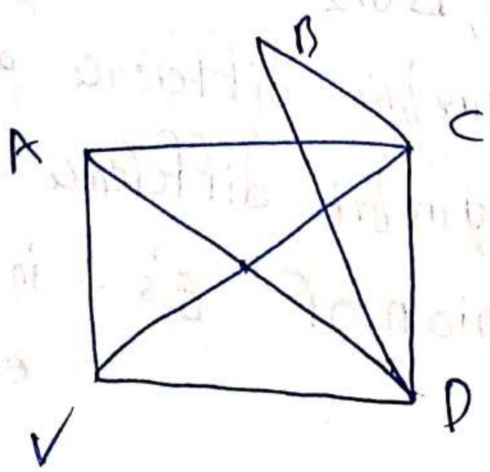
In the edge delition only delete that particular edge.



G

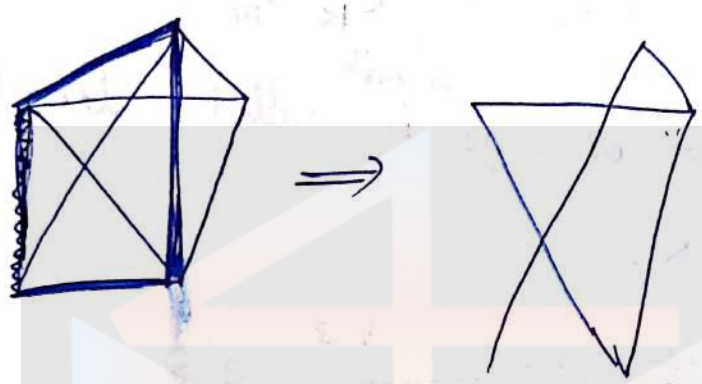


G − V



Complement of a Graph:

if G_1 is graph the Complement of a Graph $\overline{G_1}$ or $G_1' = G \Delta G_1$



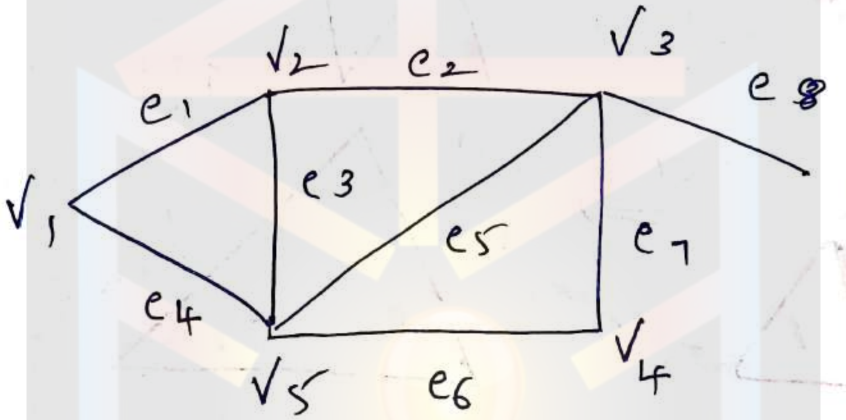
Walks & their classification:

Five important subgraphs are

i) Walk
ii) traid
iii) Circuit
iv) path
v) cycle

→ <u>Walk</u>: Graph should have atleast 1 edge altering vertices & edges of the form $V_i\, e_j\, V_{i+1}\, e_{j+1}\, V_{i+2}\, e_{j+2} \ldots\ldots\ e_k\, V_m$

The no. of edges in walk is called length of the walk.



→ $V_1\, e_1\, V_2\, e_2\, V_3\, e_8\, V_6$

Here walk length = 3 & no vertex & no edge is repeated

→ $V_1\, e_4\, V_5\, e_3\, V_2\, e_2\, V_3\, e_5\, V_5\, e_6\, V_4$

length = 5 & vertex V_5 repeated
no edge repeated

→ $V_1 e_1 V_2 e_3 V_5 e_3 V_2 e_2 V_3$

length = 4   vertex V_2 repeated
edge e_3 repeated

~~Here the length is~~

→ The initial vertex is called origin & final vertex is called terminus.

→ origin & terminus both are called terminal vertex, Non terminal vertex is called internal vertex.

→ walk begins & ends with same vertex is called closed walk

→ else open walk.

$\underline{V_1} e_1 V_2 V_3 V_5 e_4 \underline{V_1}$ (closed walk)

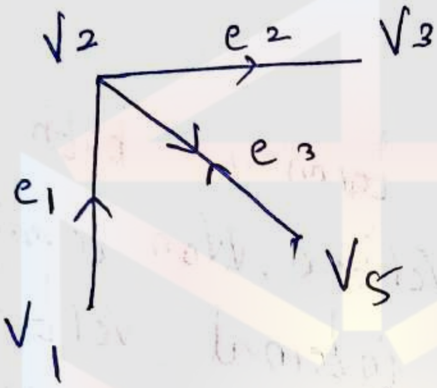
$\underline{V_1} e_1 V_2 e_2 V_3 e_5 \underline{V_5}$ (Open walk)

→ A walk from U vertex to V vertex is called UV walk

→ Trail & circuit :-

Trail is a open walk with no edge repetation.

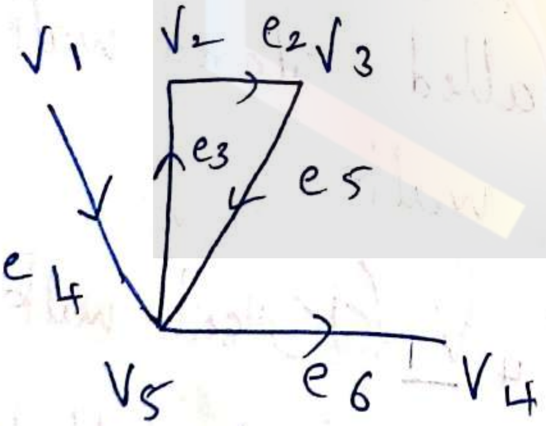
Circuit is a closed walk with no edge repetation.



$V_1 e_1 V_2 e_3 V_5 e_3 V_2 e_2 V_3$

Open E-R

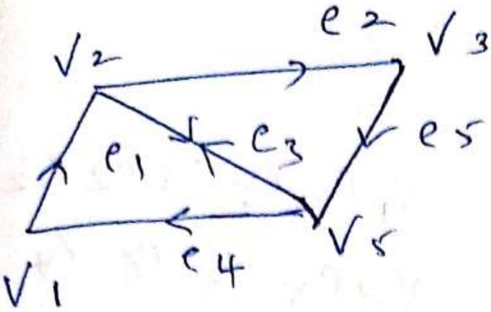
× trial



$V_1 e_4 V_5 e_3 V_2 e_2 V_3 e_5 V_5 e_6 V_4$

Open E-R ✓

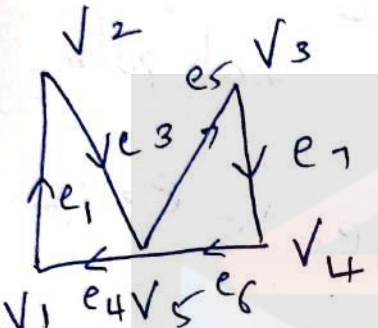
Trial



$v_1 e_1 v_2 e_3 v_5 e_3 v_2 e_2$

$v_3 e_5 v_5 e_4 v_1$

Closed E-R × Circuit



$v_1 e_1 v_2 e_3 v_5 e_5 v_3 e_7$

$v_4 e_6 v_5 e_4 v_1$ ✓

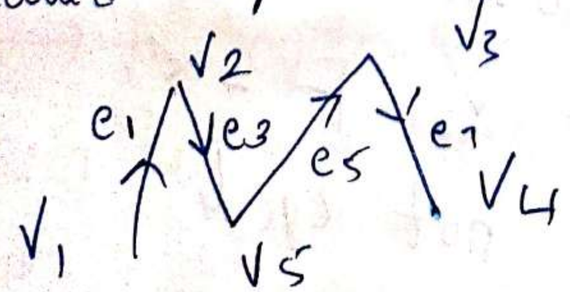
closed ✓ circuit

no edge repeat ✓

Path & Cycle:

A Trail in which no vertex repeated more than once is called path.

A circuit in which terminal vertex as an internal vertex & no internal vertex is repeated is called cycle

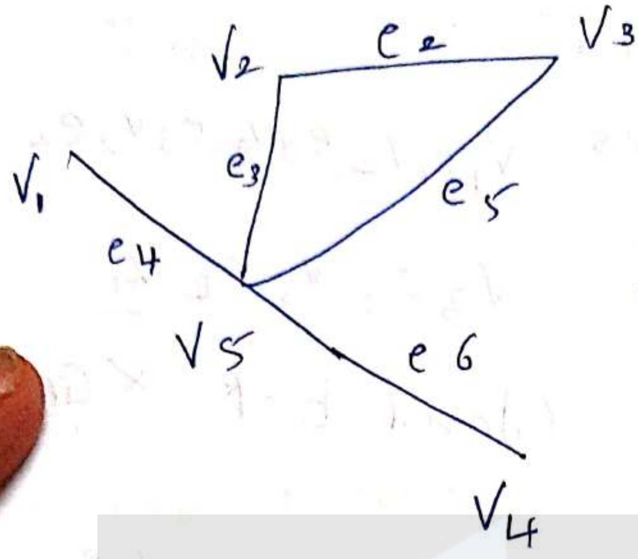


$v_1 e_1 v_2 e_3 v_5 e_5$

$v_3 e_7 v_4$

open ✓ E-×, V-×

path.

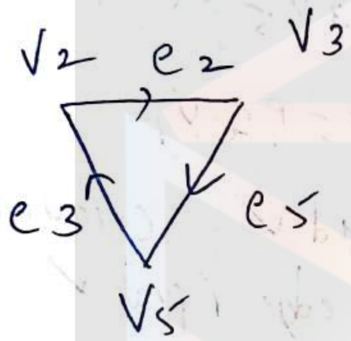


$v_1 e_4 v_5 e_3 v_2 e_2$
$v_3 e_5 v_5 e_6 v_4$

open ✓

E − ✗ V − ✓

not path.

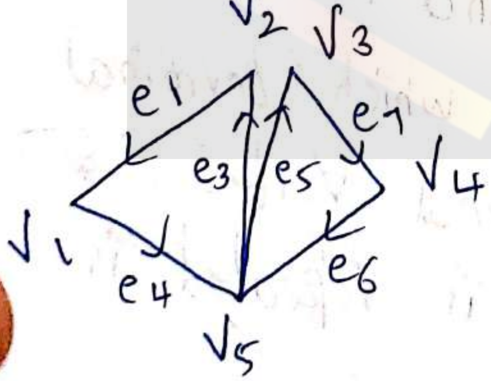


$v_2 e_2 v_3 e_5 v_5 e_3 v_2$

closed ✓

E − ✗ V − ✗

Cycle − ✓



$v_2 e_1 v_1 e_4 v_5 e_5$
$v_3 e_7 v_4 e_6 v_5 e_3 v_2$

closed ✓

V − ✓ E − ✗

not cycle

→ A walk can be a open or closed.

→ In a walk a vertex or edge can appear more than once.

→ A trial is an open walk vertex can appear more than once but an edge cannot appear more than once.

→ A circuit is a closed walk in which a vertex can appear more than once but an edge cannot appear more than once

→ A path is an open walk in which neigther a vertex nor an edge can appear more than once every path is trial but A trial need not be a path

→ A cycle is a closed walk in which neigther a vertex nor an edge can appear more than once.

Every cycle is a circuit but a circuit need not be a cycle

→ If a cycle only 1 edge loop.

→ two parallel edges form a cycle

→ In a simple graph a cycle must have atleast 3 edges (triangle)

→ In a digraph we call them as directed box, trials, circuits, paths, cycles.

→ <u>Connected & Disconnected graph</u>:

In Graph(G) order ≥ 2

then 2 vertices are said to be connected if their is atleast one path b/n one vertex to another

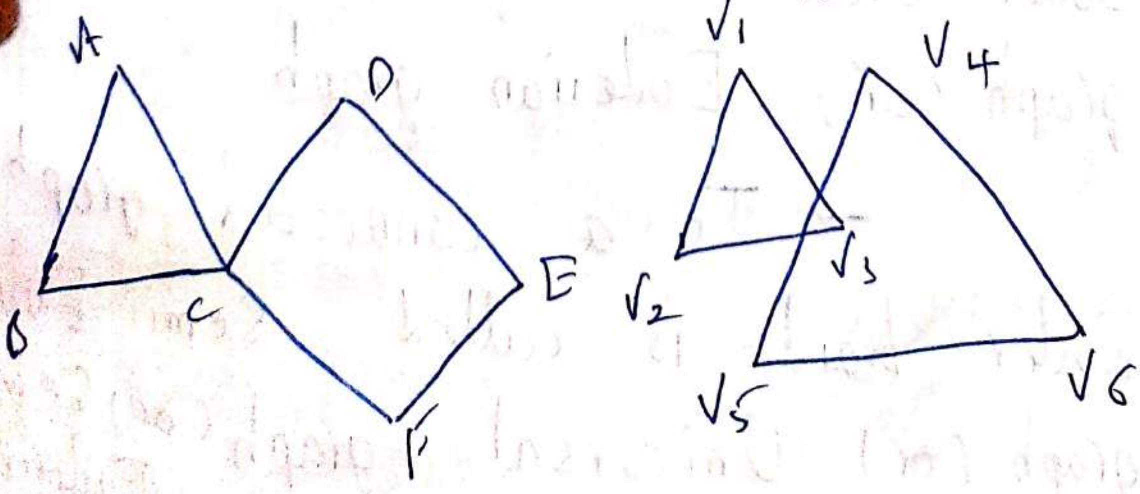
→ <u>Connected</u> if their is atleast 1 path b/n every distinct vertices in G

→ <u>Disconnected</u> if G has atleast 1 pair of distinct vertices b/n which their is no path

→ Each connected graph is a sub graph of G & is called component of G

→ In disconnected graph 2 or more components will be their

→ No. of components are represented by K(G) (In Disconnected graph in example K(G) = 2)



Euler Circuit & Euler trails :-

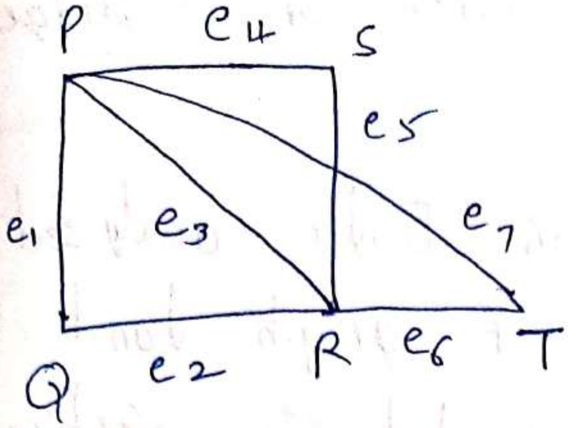
→ A circuit in Graph (G) that contain all edges of G is called Euler Circuit (or) Euler tour (or) Euelerian line

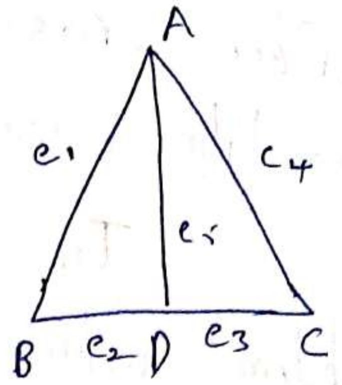
→ A trail with all edges of G is a Euler trail (or) (~~Unique~~ Unicursal line

→ In circuit or trail no edge appear more than once, a vertex can appear.

→ In a connected graph Euler circuit is called as Euler graph. (or) Eulerian graph

→ In a connected graph Euler trail is called Semi Euler graph (or) Unicursal graph (or) Semi Eulerian graph.





$Pe_1Qe_2Re_3Pe_4Se_5Re_6Te_7P$

$Ae_1Be_2De_3Ce_4Ae_5D$

Konigs berg Bride proble-

18th centuary, city name :- Konigs berg, prussia (Europe) their flowed a river named pregel river which divided the city into 4 parts 2 of these parts were banks of the river & 2 were island

These paths connected through 7 bridges by starting at any one of 4 land areas return to that are can we after crossing each of 7 bridge exactly once

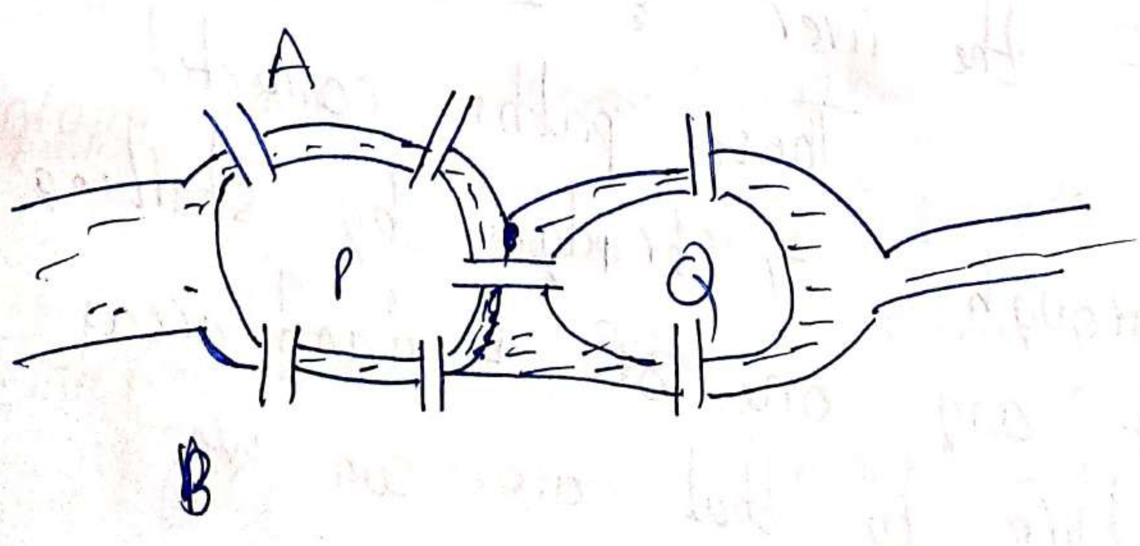
In 1736 Euler analyzed problem with help of graph land areas A, B, P, Q are taken as vertices & bridges are taken as edges.

Degree of A = Degree of B = Degree of Q = 3

Degree of P = 5

Which are not even So graph does not have euler circuit

It has no closed walk (i.e exactly once edges).

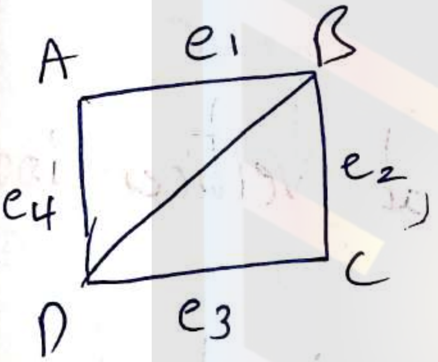


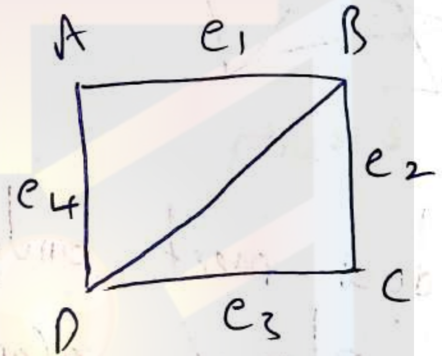
Hamilton Cycle & Hamilton path :-

A cycle contain all vertice is hamilton Cycle.

A graph contain hamil cycle is hamilton Graph

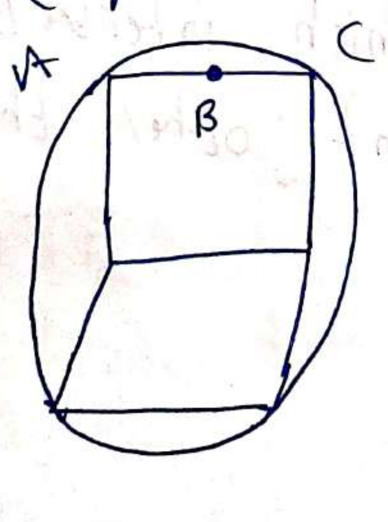
A path contain all vertice is hamilton Path



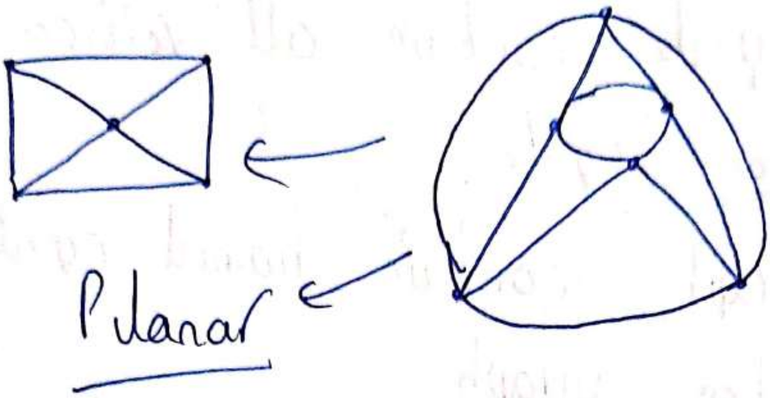


$A e_1 B e_2 C e_3 D e_4 A$ (Cycle)

$B e_1 A e_4 D e_3 C$ (Path)



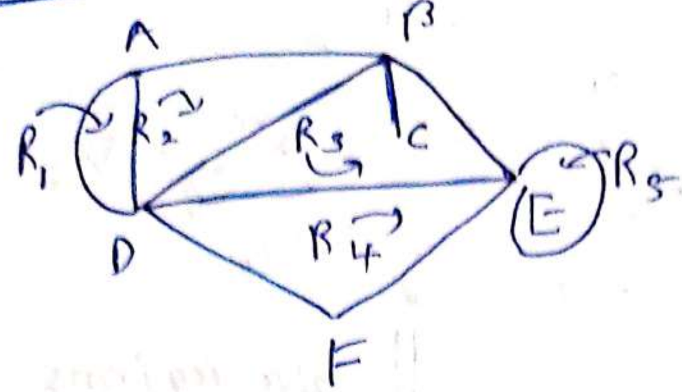
Planar & nonplanar :



if edges meet only at vertices is called planar graph

In non planar graph ~~to~~ whose every possible plane drawing contains atleast 2 edges which intersect each other at point other than vertices

Euler's Formula :



$deg(R_1) = 2$ $\quad$ $deg(R_4) = 3$
$deg(R_2) = 3$ $\quad$ $deg(R_5) = 1$
$deg(R_3) = 4+1$ $\quad$ $deg(R_6) = 6$

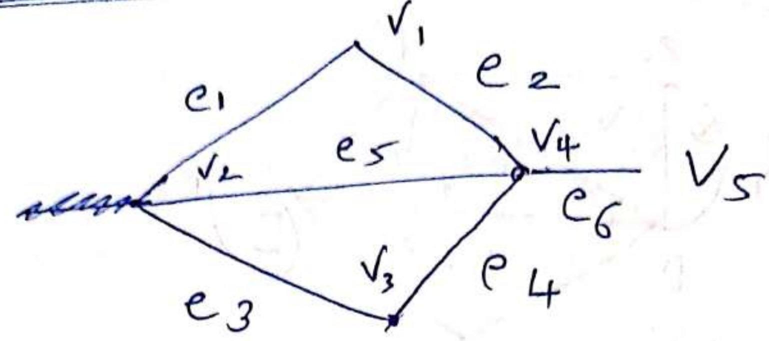
Diagram devides plane into no. of parts called regions or phases

No. of edges that form the boundary of a region is called the degree of that region.

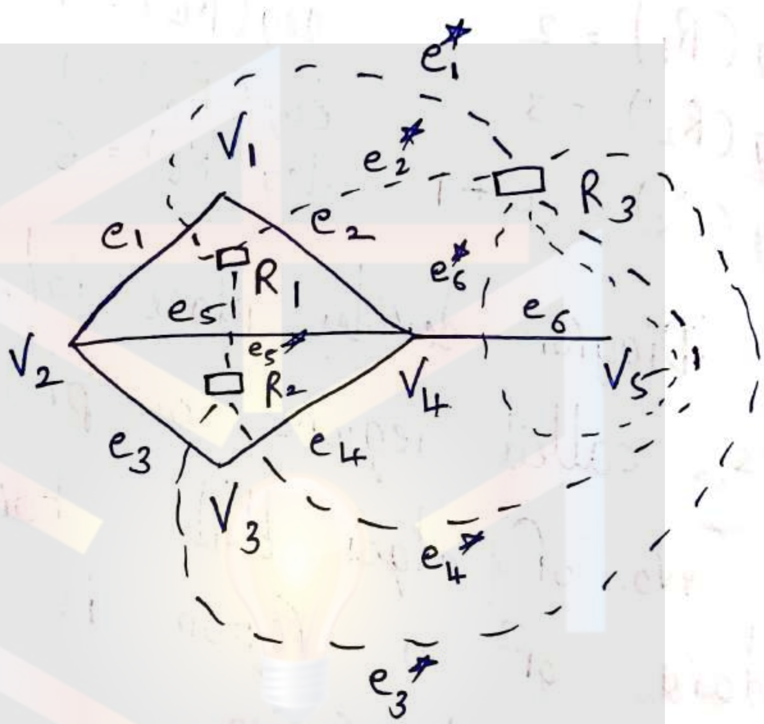
It follow~~ing~~ & the hand ~~some~~ sum of the degree = 2 × no. of edges

If their are m edges & n vertices then no. of regions = $m - n + 2$

Dual of Planar graph :-



⇓ give regions



→ Choice 1 point inside each region R_1, R_2, R_3 (V_1^*, V_2^*, V_3^*)

→ If 2 regions are adjacent connect e_k^* b/n V_i^* & V_j^*

→ G^* is Geometric dual of G

→ Their is 1 to 1 b/n G & G^*

→ Pendent vertex become loop

→ loop become pendent

→ serial edges becomes parallel

→ parallel " " serial

→ no. of edges in region = degree of vertex.

→ G & G^* are planar graph.

→ $n^* = R$, $m^* = M$, $R^* = N$

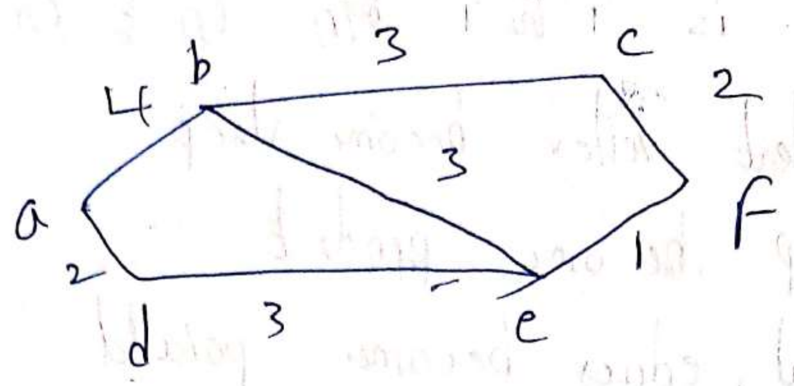
n^* = vertices,

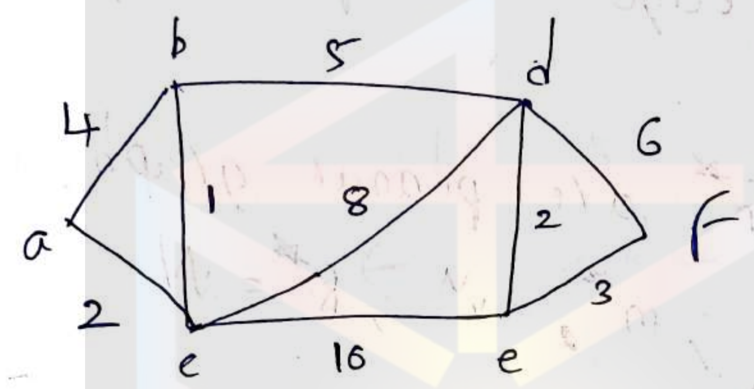
m^* = edges

R^* = region

Shortest path algorithm:

Weighted graph: IF graph have number assigned to the edges is called weighted graph.





a - c - b - d - e - 2

Degree Sequence:

Non increasing older of degree's is called degree Sequence

deg(A) = 1

(B) = 3

(C) = 2

(D) = 2

(E) = 2

3, 2, 2, 2, 1

Matrix Representation:

Icidence matrix

If G be a graph with n vertices e edges & without selfloops then incidence matrix is e, $A(G) = [a_{ij}]$ defined by

$$[A_{ij}] = \begin{cases} 1 & (\text{if vertex } v_i \text{ is incident on edge } e_j) \\ 0 & (\text{otherwise}) \end{cases}$$

In matrix their is one row for each ~~worte~~ vertex & one column for each edge.

Elements of incidence matrix or either 0 or 1.

It is also known as binary matrix

Properties

→ Every edge of a graph incident on exactly 2 vertices

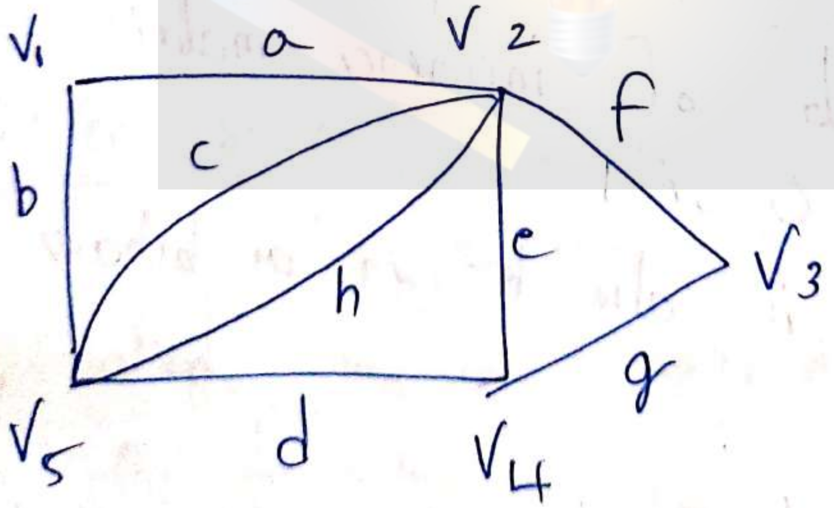
→ Sum of ones in each row = degree of vertex

→ A row with all zero's is called isolated vertex.

→ Parallel edges produce identical column c & h

→ 2 graphs are isomorphic if & only if their matrix differ only by ~~Re~~ Permutation of rows & columns.

→



If graph is disconnected it consists of 2 components G_1 & G_2

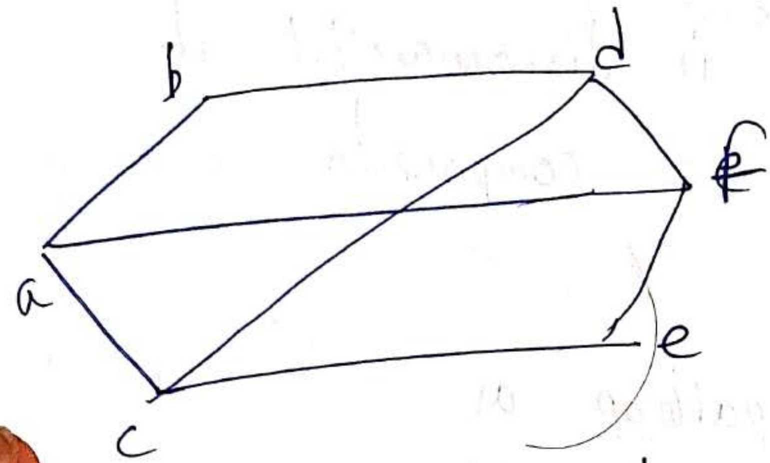
Incident partition as

$$A(G) \left[\begin{array}{c|c} A(G_1) & 0 \\ \hline 0 & A(G_2) \end{array}\right]$$

Let $A(G)$ be incidence matrix of a connected graph G of n vertices

rank of $A(G) = n-1$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}$$



$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & f \\
\hline
a & 0 & 1 & 1 & 0 & 0 & 1 \\
b & 1 & 0 & 0 & 1 & 0 & 0 \\
c & 1 & 0 & 0 & 1 & 1 & 0 \\
d & 0 & 1 & 1 & 0 & 0 & 1 \\
e & 0 & 0 & 1 & 0 & 0 & 1 \\
f & 1 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

Adjacency matrix

If G be a graph with n vertices & without parallel edges The Adjacency matrix is a n/n matrix defined by

$X(G) = [x_{ij}]$

$[x_{ij}] = \begin{cases} 1 & \text{(if their is a edge b/n } i^{th} \text{ \& } j^{th} \text{ vertices 0 otherwise)} \\ 0 & \text{(otherwise} \end{cases}$

Properties

→ If diagonal entries are zero's their is no self loop

→ No parallel edges

→ Sum of entrie in row or column = degree

→ It is a symmetric matrix

$[x_{ij}] = [X_{ji}]$

→ If two rows are interchange automatically it's column by me interchanged.

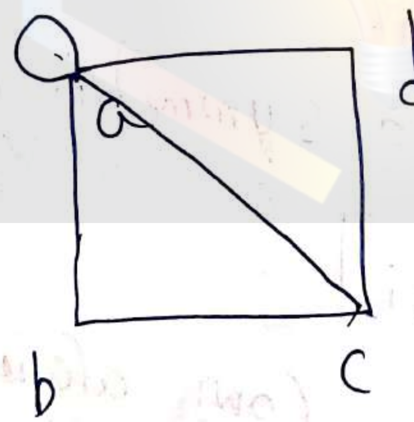
$$X(G_2) = R^{-1} X(G_1) R.$$

→ In a disconnected graph

$$X(G) = \left[\begin{array}{c|c} X(H_1) & 0 \\ \hline 0 & X(H_2) \end{array}\right]$$

→ we can construct a graph of n vertices without parallel edges from a square is symmetric binary $n \times n$ matrix

Power of Adjacency Matrix:



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$X^2(G) = X(G) \cdot X(G)$$

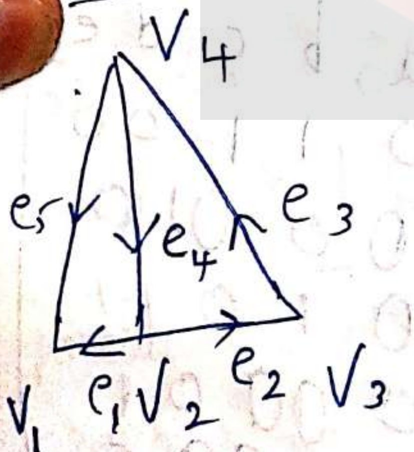
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 3 & 2 \\ 2 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \\ 2 & 2 & 1 & 2 \end{bmatrix}$$

$$X^3(G)$$

$$\begin{bmatrix} 11 & 7 & 8 & 7 \\ 7 & 3 & 6 & 3 \\ 8 & 6 & 5 & 6 \\ 7 & 3 & 6 & 3 \end{bmatrix}$$

$$X^n = X^{n-1} \cdot X$$

Insidence Matrix for Digraph



$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ \forall \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 & 0 \\ 0 & -1 & +1 & 0 & 0 \\ \cancel{0} & \cancel{1} & \cancel{-1} & \cancel{0} & \cancel{0} \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

incomming = -1
out going = 1

$[A_{ij}] = 1$ if v_i is initial vertex of edge e_j

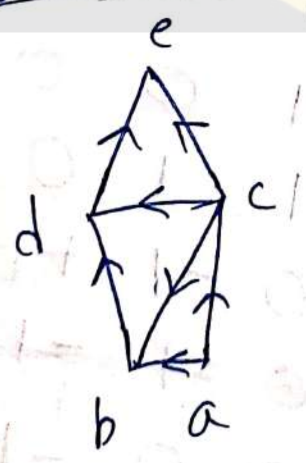
-1 if v_i is terminal vertex of e_j

0 if v_i is not incident on v_i

(-1 incomming
1 outgoing)

<u>Note</u>: Sum of all column $= 0$

<u>Adjacency matrix</u>



$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Let D be a digraph without parallel edges the adjacency matrix is n×n matrix

$$X(D) = [x_{ij}]$$

$$x_{ij} = \begin{cases} 1 & \text{if their is an edge directed from } i^{th} \text{ vertex to } j^{th} \text{ vertex} \\ 0 & \text{otherwise} \end{cases}$$

sum of rows = outdegree
sum of column = indegree
sum of nonzero element = no. of edges

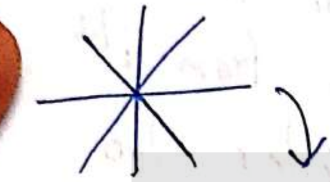
D^R obtained by reversing the direction of edge.

→ A graph with self loop & no parallel edges is called psudo graph

→ A graph with one vertex is called trivial graph,

→ No. of edges in a complete graph of n vertices → $\frac{n(n-1)}{2}$ edges.

$(K_{1,n})$ complete graph notation



$(K_{1,n})$ is a star graph

→ 2 edges adjacent if they are incident on common vertex

→ loop is counted twice while calculating the degree = valency

→ $\delta(G)$ = min $\Delta(G)$ = max

min of all degree's   max of all degree's

→ In k regular graph, min degree = max degree = k.

edges

are

of

ce?

→ ring sum = ~~even delta e_2~~

$e_1 \Delta e_2$ or $e_1 \oplus e_2$

→ By deleting all self loops & for every pair of adjacent vertices make one edge joining them from G we obtain simple ~~s~~ spanning subgraph (G)

$G \oplus H = G - H$

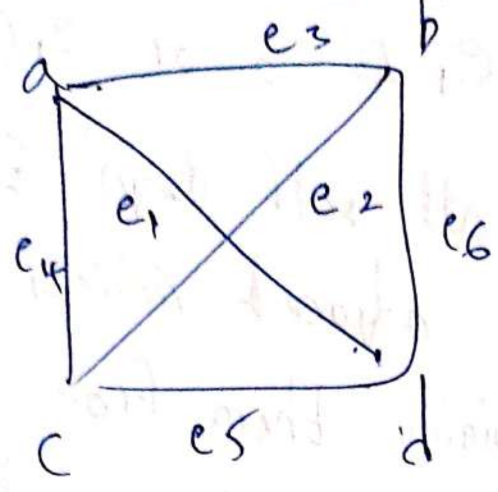
→ Ring sum can be represented with

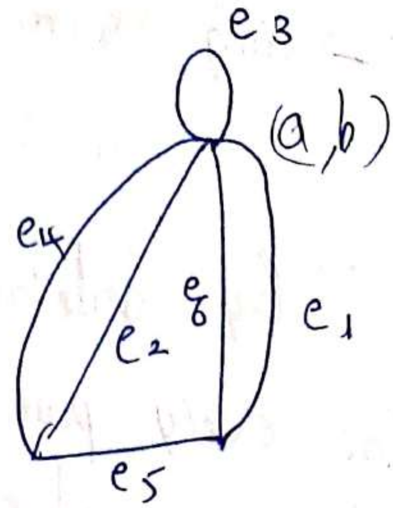
$\oplus$ XOR

If G is a graph H is a subgraph ring sum of G, H =

$G \oplus H$ $(G - H)$

Merging of vertices if 2 vertices are replaced by 1 vertex it such a manner that edges incident on both vertices are incident on new vertex.





→ Sometimes a walk in a graph contain a path

→ a self loop itself is a circuit

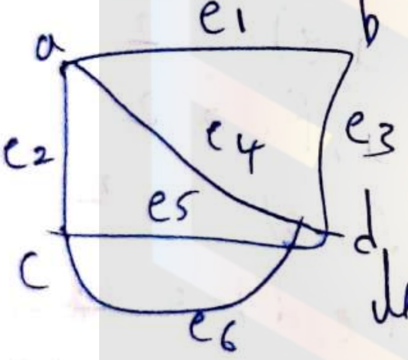
→ A connected graph G remain connected after removing a edge from G if edge is in some circuit

→ In a disconnected graph G if their are exactly 2 vertices of odd degree then they must be in some component

→ A simple graph with n vertices & k components can have atmost $(n-k)(n-k+1)/2$ edges.

→ The minimum weight of (u,v) path denoted by $d(u,v)$
↓
distance

→ <u>Distance is</u>



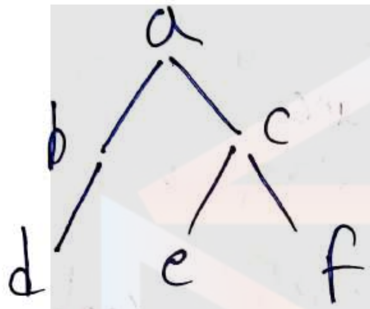
If G is a graph x & y are any vertices length of the shortest path b/n x & y is called $d(x,y)$

$a e_1 b \Rightarrow 1$

Shortest path is $a e_1 b$

distance of $a,b = 1$

Finding ditanca b/n 2 velbica of a trea is easy coz their is no circuit & their for their is only one path b/n every pair of velbica.



$d(a,b) = 1$

$d(a,d) = 2$

$d(a,c) = 1$

$d(c,d) = 3$

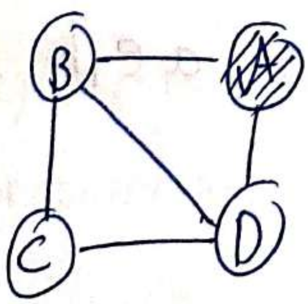
$d(a,e) = 2$

$d(a,f) = 2$

$d(c,b) = 2$

$d(e,d) = 4$

Eccentricity:



$d(A,A) = 0$

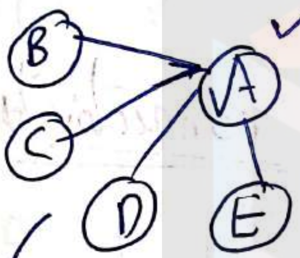
$d(A,B) = 1$

$d(A,C) = 2$ ✓

$d(A,D) = 1$

Let G be a graph & x be any vertex of A then the length of largest path starting from vertex x denoted by $E(x)$ = maximum $\{d(x,y) = y \in G\}$

Center

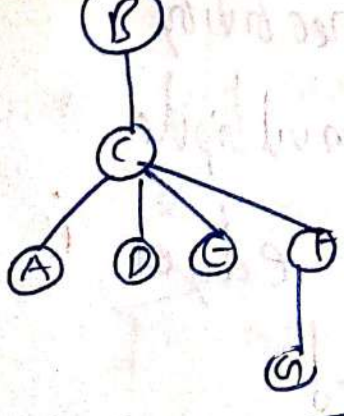


Here eccentricity is minimum

ex: school is at center of town will reduce the distance of buses to travel

A is center

Diameter & radius:



Diameter = 3 ($A \to C \to F \to G$)

radius = 2 (any)

diameter Highest eccentricity of a vertex
maximum distance b/n pair of vertices

$$d = max\{E(V) : V \in G\}$$

Radius is minimum among all maximum distances b/n a vertex to all other vertex

Cut vertices, edges & connectivity

Connectivity A graph is said to be connected if their is a path b/n every pair of vertices, from every vertex to ~~anot~~ any other vertex their should be some path to traverse called connectivity

A graph with multiple disconnected vertices & edges is said to be disconnected

It is 2 types

i) Vertex connectivity

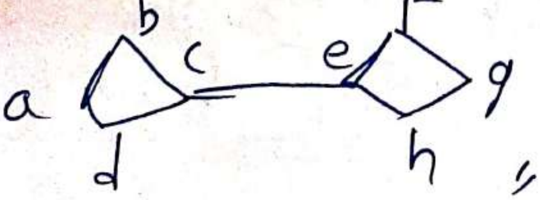
ii) edge connectivity

<u>Cut vertex:</u> Let G be a connected graph A vertex v belongs to G is called cut vertex of G if G - V results disconnected graph

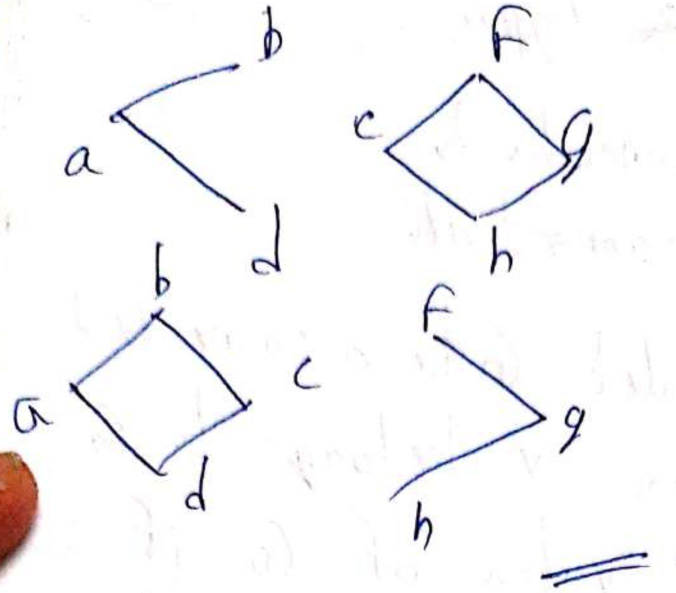
Removing a cut vertex from a graph breaks it into 2 or more graphs.

<u>Note:</u> A connected graph G may have max n - 2 cut vertices

In the graph by removing e or c vertices the graph will become disconnected



$\Rightarrow$

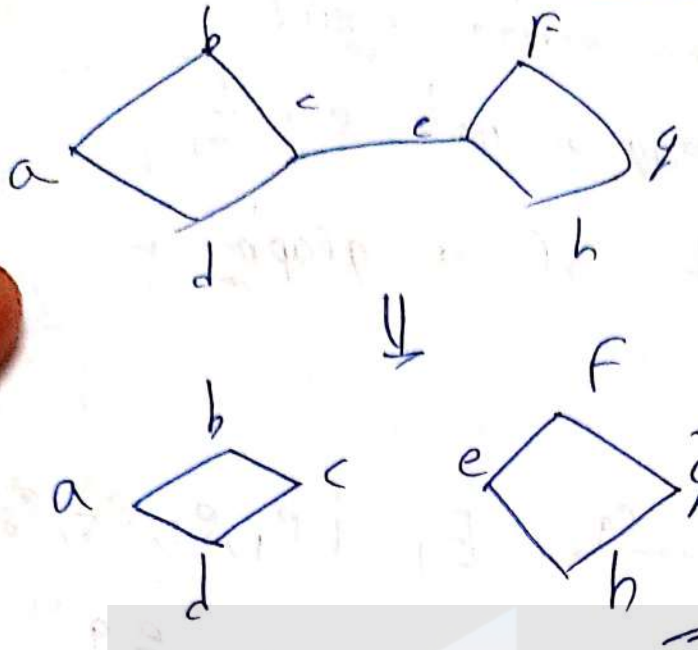


<u>Cut edge</u> (Bridge) :

Let G be a connected graph the edge $e \in G$ is called cut edge if $G - e$ results in a disconnected graph.

If removing an edge in graph results into 2 or more graph then that edge is called cut edge

ex $\{c, e\}$



→ Let G' be a connected graph with n vertices then a cut edge $e \in G$ if & only if e is not a part of any cycle in G

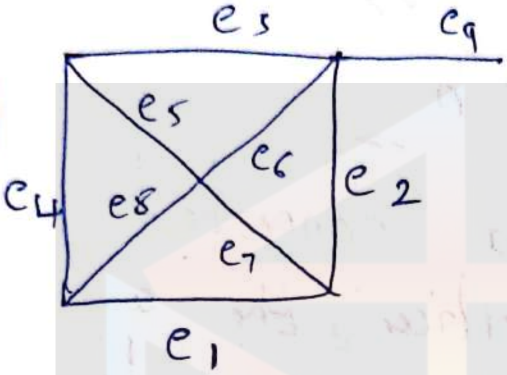
→ maximum no. of cut edges possible is $n-1$

→ Whenever cut edge is exist cut vertices also exist coz at least one ~~ed~~ vertex of cut edge is a cut vertex.

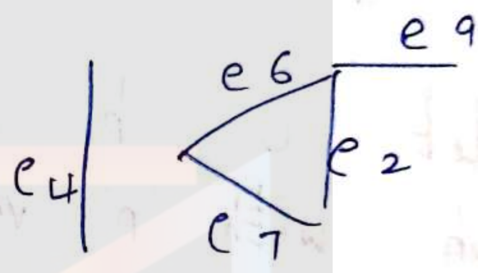
if cut vertex exist cut edge may or not exist

~~cut set of a graph~~

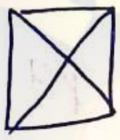
Cut set:



$E_1 = \{e_1, e_3, e_5, e_8\}$



$E_2 = \{e_9\}$



Let $G = (V, e)$ be a connected graph a sub set $\bar{e}$ of e is called a cut set of G if deletion of all edges $\bar{e}$ from G makes G disconnected

if deleting a certain no. of edges from a graph makes it disconnected the those deleted edges are called cut set of graph

<u>Block</u> a connected graph G is said to be separable if its vertex connectivity is one i.e the separable graph can be decomposed into 2 or more than 2 non separable graphs.

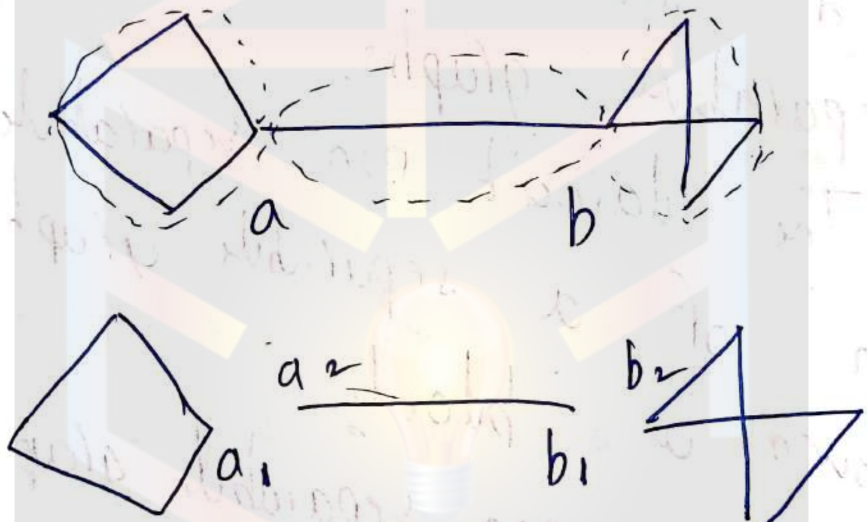
The largest non separable sub graph of a separable graph is known as block

every non separable graph consist of only one block i.e graph itself

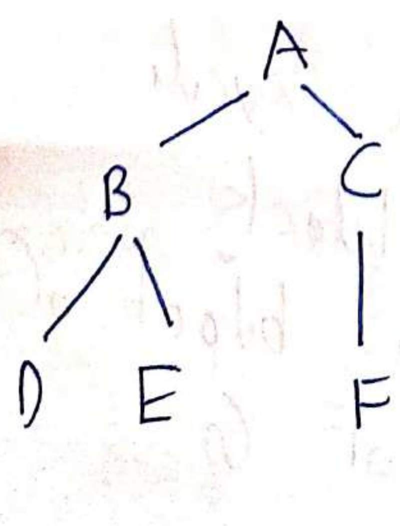
an edge of a cycle can't itself be a block

An edge is a block of G if it is a bridge of G

→ the blocks of tree are its edges. A family of paths in a graph G is said to be internally disjoined if no vertex of G is an internal vertex of more than 1 path of the family.



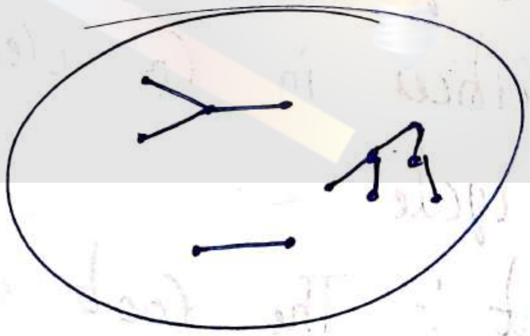
Trees



A tree is a connected undirected graph with no simple circuit (no cycles)

An undirected graph is a tree if & only if their is a unique simple path between any two of its vertices

Forest forest is a graph with 3 connected components shown below. It is a collection of tree's



In a tree their is one & only one path b/n every pair of vertice tree with n vertices has n-1 edges

→ In a rooted tree in which one vertex has been designated as root & every edge is directed away from root.

→ root indegree = 0

→ In degree of all other nodes = 1

→ A connected graph is said to be minimally connected if the removal of any one edge from it disconnect the graph

→ A connected graph G is a tree if & only if adding an edge b/n any 2 vertices in G creates exactly one cycle.

→ Parent :- The root above the child is "

→ Child :- The nodes below parent are child

→ Siddings: The child have the same parent.

→ leaf node: No children.

& out degree = 0

root - v

leaf node V_5, V_6, V_8, V_9

V_1 - ancenster

~~desent~~ dsenendent

V_3 V_6 V_5

V_1 Parent

children V_3

sibli

V_7, V_2

V_5 V_6

V_8 V_9

m-ary tree :-

A rooted tree t is called m-ary tree if every vertex t is of Out degree less than or equal to m

complete m-ary tree if every internal vertex t is of out degree m

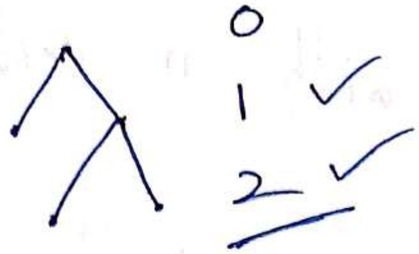
binary tree if every internal vertex t is of outdegree 1 or 2

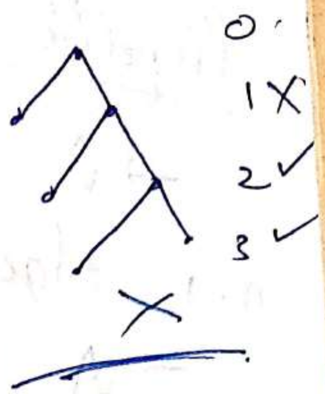
i.e if every vertex at most 2 children

Full Binary tree if all the leaves in tree are at level h

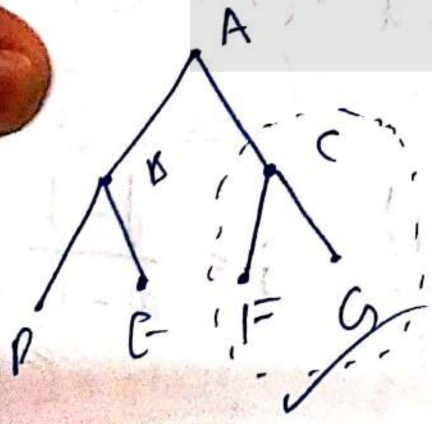
Balanced tree: A rooted tree of height H is said to be balanced if the level no. of every

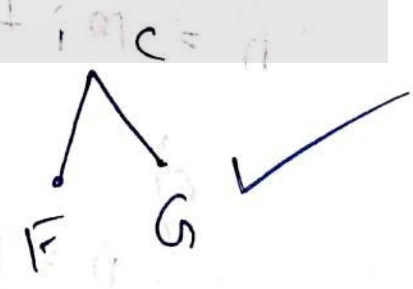
Leaf is h or h-1





<u>Subtree</u>: If a is a vertex in a tree the subtree (Sabutree) with a as it's root is a subgraph of the tree consisting of a and it's decendents & all edges & insendent to those decendents





Properties of trees:

→ A tree with n vertices has n-1 edges

→ A full mary tree with i internal vertices contain $n = mi + 1$

→ A full mary tree with

a) m vertices has $i = \frac{(n-1)}{m}$

no. internal vertices

$l = \frac{[(m-1)n + 1]}{m}$

no. leaves l

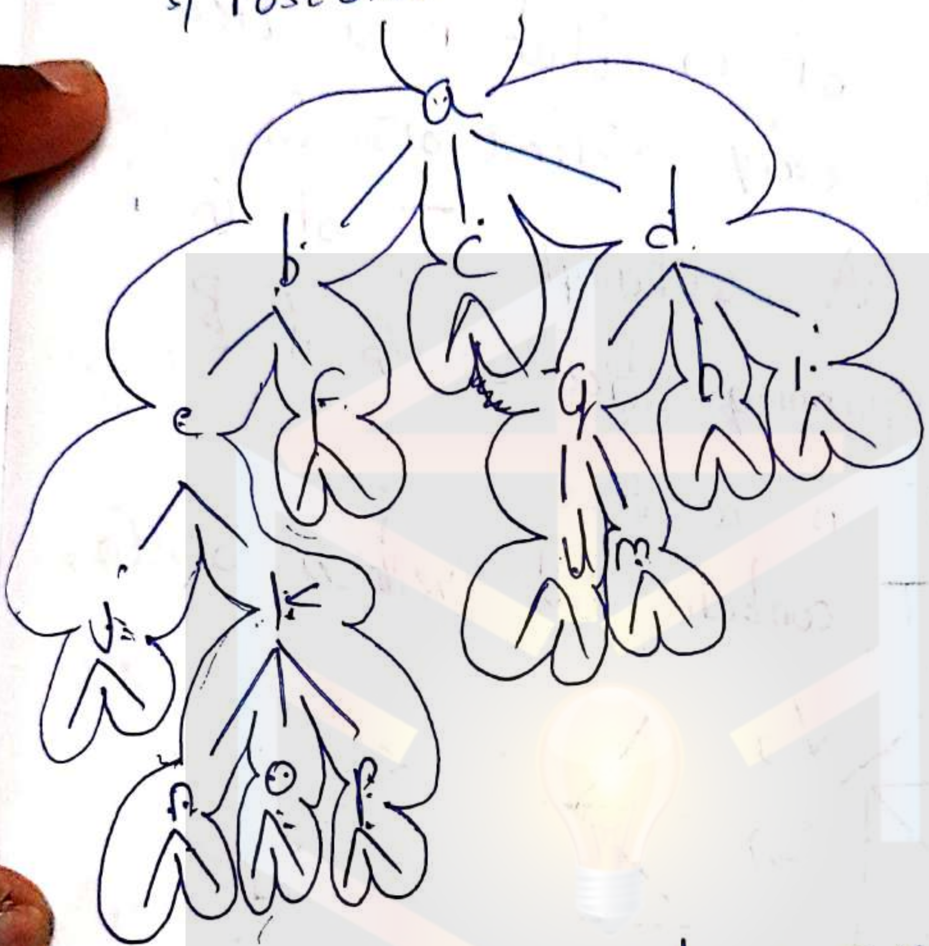
b) i internal vertices are

their $n = mi + 1$, $l = (mi - 1)$

c) $n = \frac{(ml - 1)}{n - 1}$ $i = \frac{l - 1}{m - 1}$

Tree Traversal :

1) Preorder — root, left, right
2) Inorder — left, root, right
3) Postorder — left, right, root



Preorder : a b e j k n o p f c d g l m h i

Inorder : J e n k o p b f a c l g m d h i

Postorder : J n o p k e f b c l m g h i d a

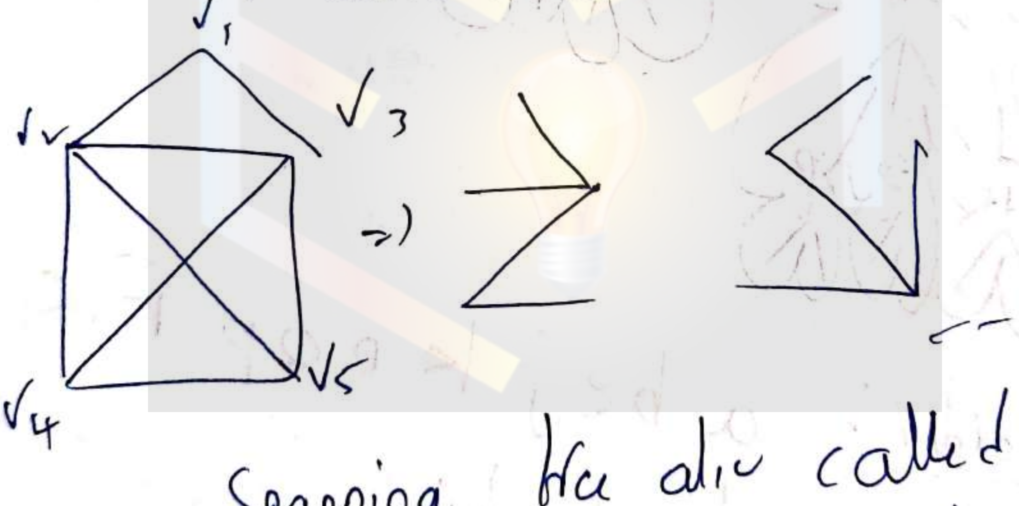
Spanning tree:

Let G be a simple graph a spanning tree of G is a subgraph of G that is a tree containing every vertex of G

A subgraph T of G is called spanning tree of G if

→ T is a tree

→ T contain all vertices of G



Spanning tree also called a maximal tree or skeleton or scaffolding edges of spanning tree called branches edges of G that are not in T ($\overline{T}$) called chords

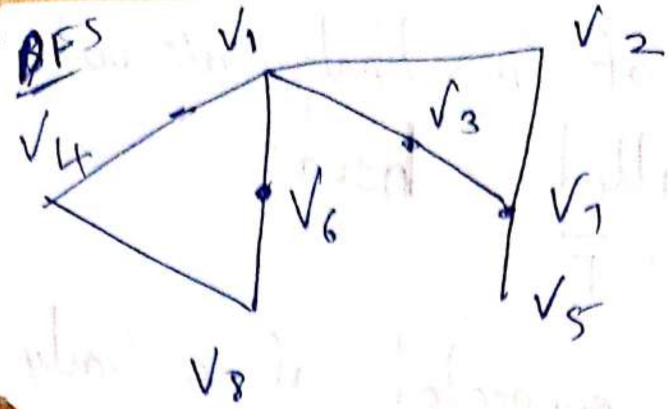
$G = T \cup \overline{T}$

Graph is connected if & only if it has a spanning tree

Spanning trees together called spanning forest

If there are n vertices m edges have $n-1$ branches $m-n+1$ chords

DFS



DFS
→ root
→ Small subroot
→ Back tracking

$V_1 - \{ V_2 \; V_3 \; V_4 \; V_6 \}$

$V_2 - \{ \cancel{V_1}, V_4 \}$

$V_7 - \{ V_5, V_3, \cancel{V_2} \}$

$V_3 - \{ V_1, \cancel{V_7} \}$

$V_7 - \{ V_5, \cancel{V_3}, \cancel{V_2} \}$

$V_5 - \{ \cancel{V_7} \}$

$V_7 - \{ \cancel{V_3}, \cancel{V_2} \}$

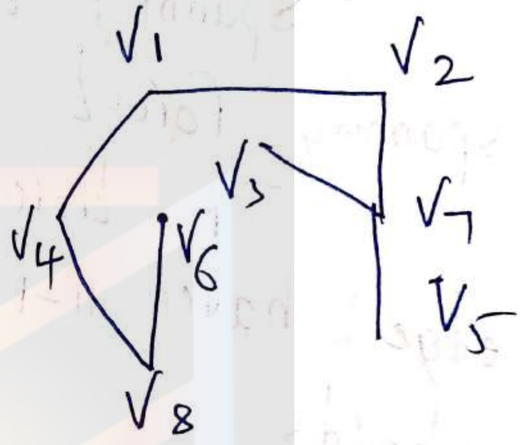
$V_2 - \{ \cancel{V_1}, \cancel{V_7} \}$

$V_1 - \{ \cancel{V_2} \; \cancel{V_3} \; V_4 \; V_6 \}$

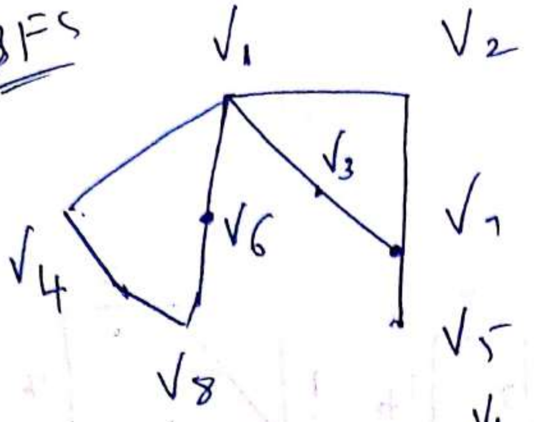
$V_4 - \{ \cancel{V_1} \; V_8 \}$

$V_8 - \{ \cancel{V_4}, V_6 \}$

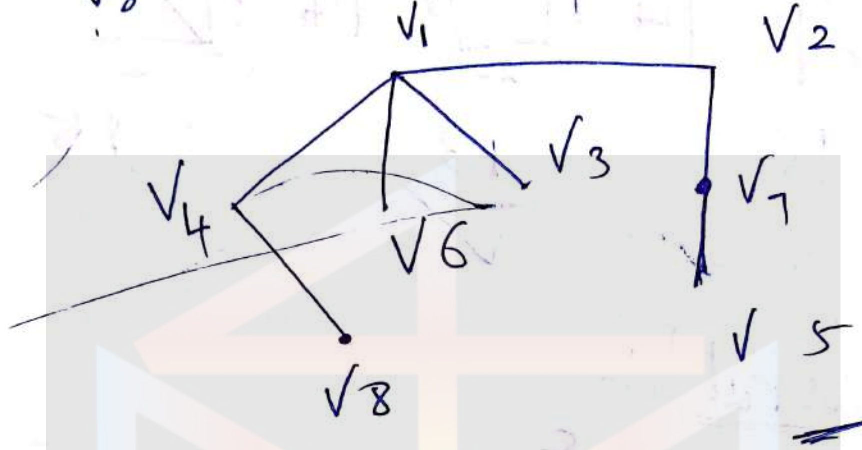
$V_6 - \{ V_1, \cancel{V_8} \}$



BFS



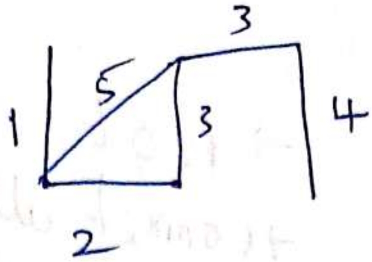
→ root
→ connect all
→ Back track

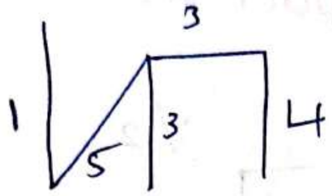


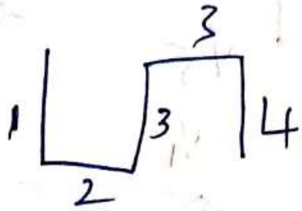
Minimal Spanning tree

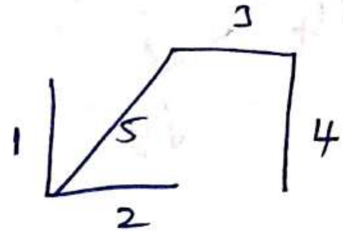
A minimal spanning tree in a connected weighted graph is a spanning tree that has the smallest possible sum of weight of its edges

Weighted graph, A possitive real number associated with each edge



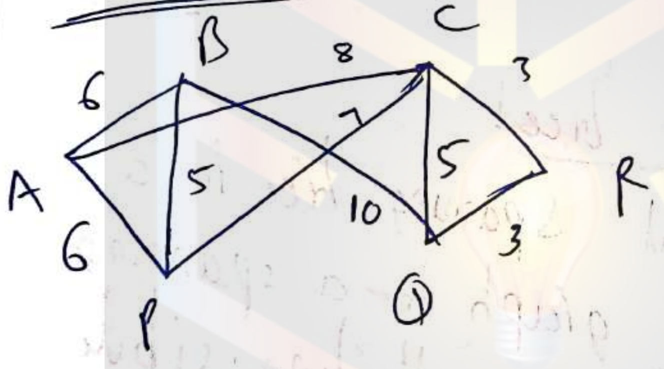






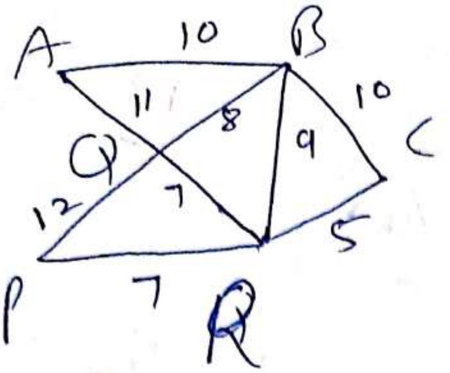
16 13 ✓ 15

Kroskal's



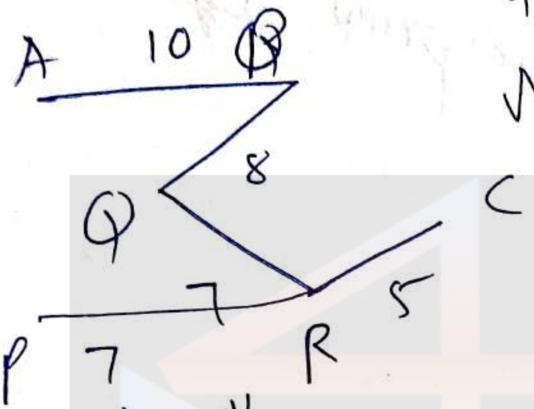
CR	QR	CQ	BP	AP	AB	PC	AC
3	3	5	5	6	6	7	7
X	Y	W	Y	W			

BQ
10



CR	QR	PR	BQ
5	7	7	8
Y	Y	Y	Y

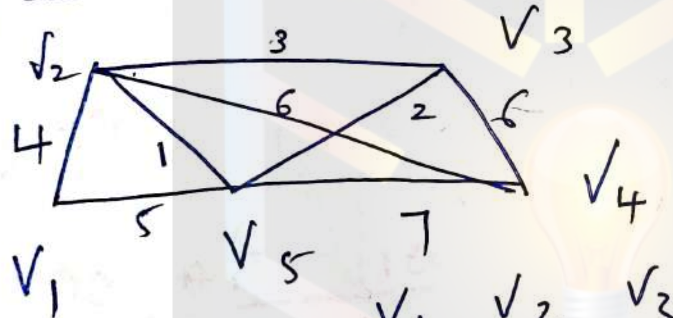
BR	BC	AB	AR	PQ
9	10	10	11	12
N	N	Y	N	N



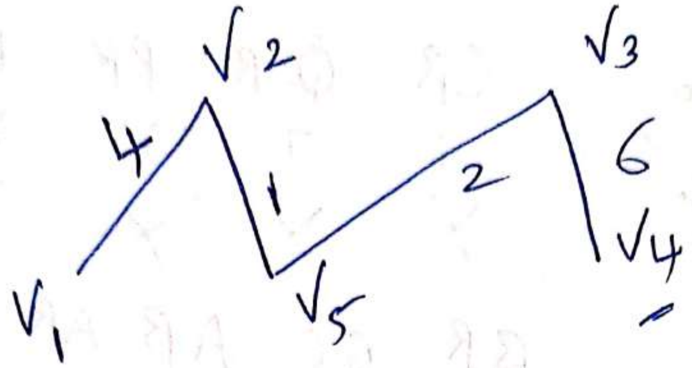
$5+7+7+8+10$

$\Rightarrow 37$

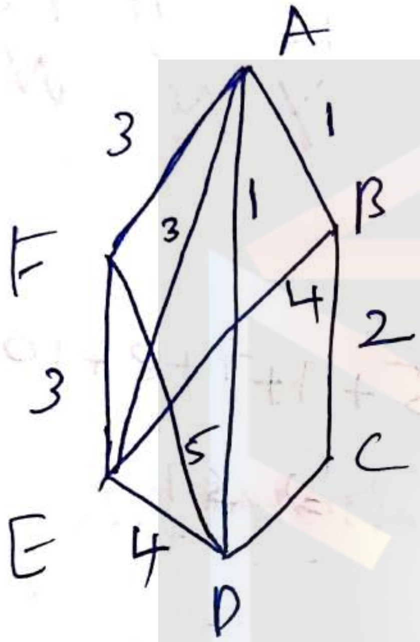
Prim's algorithm



	V_1	V_2	V_3	V_4	V_5
V_1	~~∞~~	4	∞	∞	5
V_2	4	—	3	6	1
V_3	∞	3	—	6	2
V_4	∞	6	6	—	7
V_5	5	1	2	7	—



13



Use prims algorithm

Characterizations:

→ T is a tree

→ T contains no cycles has n-1 edges

→ T is connected

→ Every edge is cutedge.

→ Any 2 vertices of tree are connected by exactly one path

→ For any new edge e the graph T+e has exactly 1 cycle.

no. of tree's:

Kirchoff's theorem:

→ It is usefull to find no. of spanning tree's that can be formed from a connected graph

Step 1: If their is an edge from vertex place 1 in that row. oth 0

2: replace diagonal with degree of that vertex

3: replace non diagonal 1's with -1

4: leave the first row & column & find co-factor or determinant or m

Application of tree's:

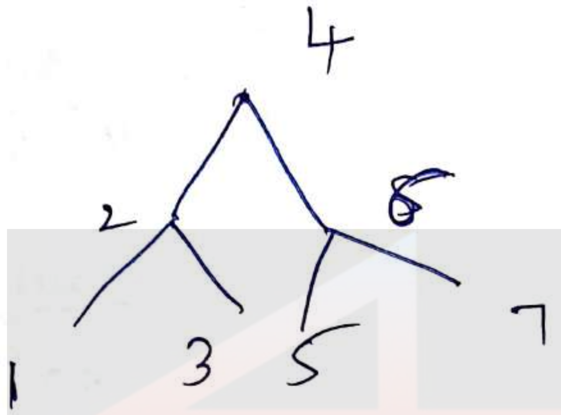
Huffman coding

rate 11 000

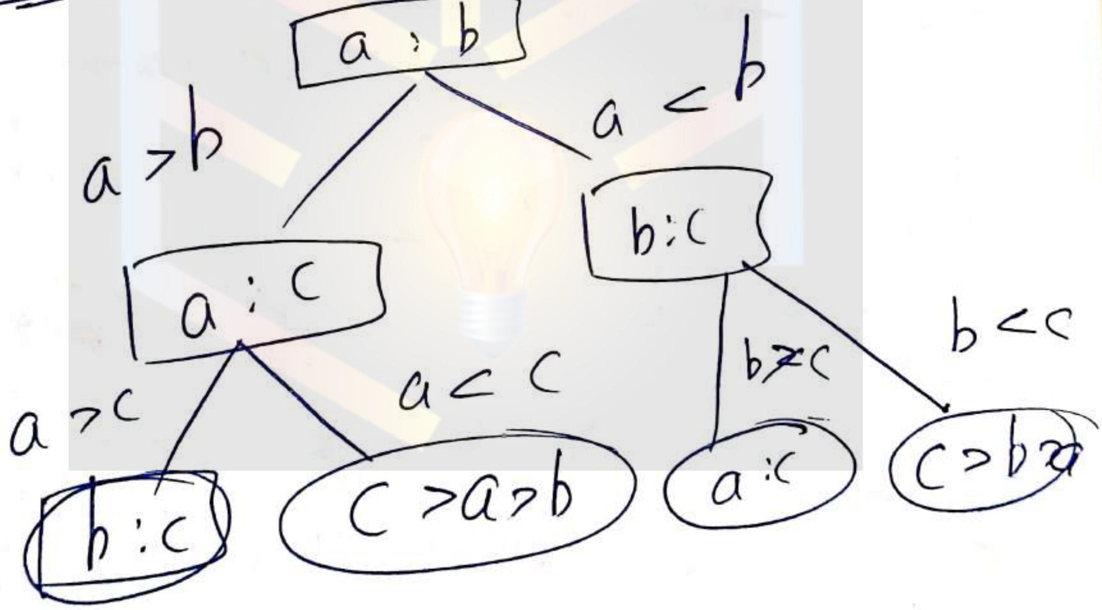
001 000 011 10 0 001 10

Binary search tree:

It is a tree in which left element is small & right element is big



Desicion tree



Game Well